Utilisation des symétries pour l'accélération du code potentiel linéaire Nemoh.

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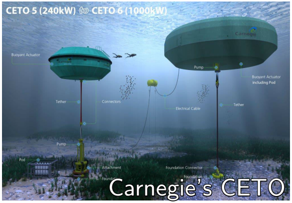
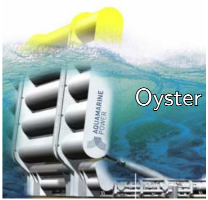
Summary

Linear potential flow simulations
of symmetric Wave Energy Converters (WECs)
with the open source code Nemoh.

Axisymmetric shape



Cylindrical or prismatic shape



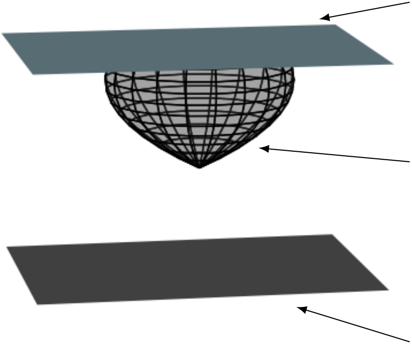
Regular array

. . .

Linear potential flow theory

$$u = \nabla\phi, \qquad \phi = \mathrm{Re}\left(\Phi e^{-i\omega t}\right)$$

$$g\frac{\partial\Phi}{\partial z} - \omega^2\Phi = 0$$



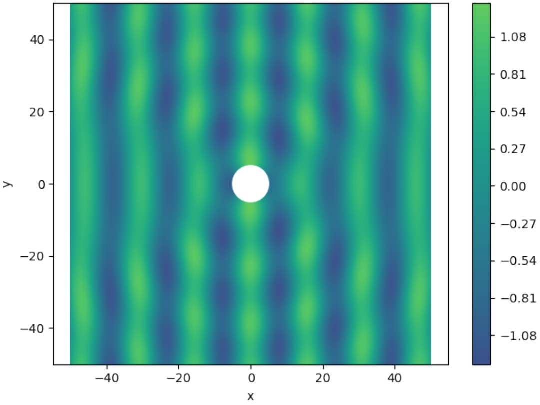
$$\nabla^2\Phi = 0$$

$$\nabla\Phi \cdot n = u \cdot n$$

$$\frac{\partial\Phi}{\partial z} = 0$$

Example of result

Free surface elevation of a diffracted wave around a floating sphere



Also: hydrodynamic coefficients (added mass, radiation damping), ...

Boundary integral formulation

Introducing the Green function $G(x, .)$, solution of the problem for a singularity in x and for the boundary conditions of the free surface and sea bottom.

Potential at the body boundary (output)

$$\Phi_i = \sum_j \overbrace{\left(\iint_{\Gamma_j} G(x_i, y) \, \mathrm{dS}(y) \right)}^{S_{ij}} \sigma_j$$

$$(u \cdot n)_i = \frac{\sigma_i}{2} + \sum_j \underbrace{\left(\iint_{\Gamma_j} (\nabla_x G(x_i, y) \cdot n) \, \mathrm{dS}(y) \right)}_{V_{ij}} \sigma_j$$

Normal velocities (input)

Boundary Elements Method (BEM) solver:

1. Evaluate the matrices S and V;
2. Solve the linear system $u = \left(\frac{\mathbb{I}}{2} + V\right)\sigma$;
3. Deduce $\Phi = S\sigma$.

   + post-processing

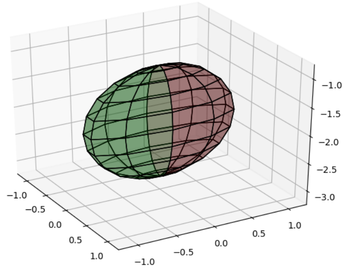
Nemoh

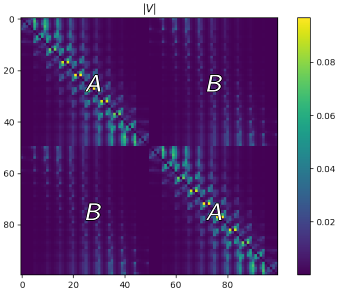
Open source Boundary Elements Method solver
released in 2014 by École Centrale Nantes.
`https://lheea.ec-nantes.fr/logiciels-et-brevets/nemoh-presentation-192863.kjsp`

Following work has been done in my own refactored development version:
`https://github.com/mancellin/capytaine`

One vertical symmetry plane

Block symmetric structure



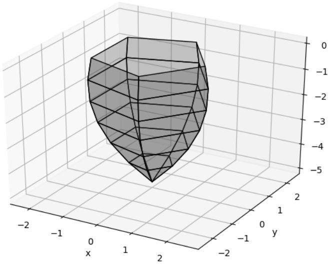


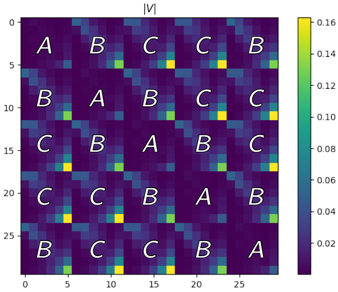
- Computation of only half of the coefficients.
- Faster resolution of the linear system:

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} \sigma \\ \sigma' \end{pmatrix} = \begin{pmatrix} u \\ u' \end{pmatrix} \quad \Leftrightarrow \quad \begin{cases} (A+B)(\sigma+\sigma') = (u+u') \\ (A-B)(\sigma-\sigma') = (u-u') \end{cases}$$

Axial symmetry

Block symmetric circulant structure

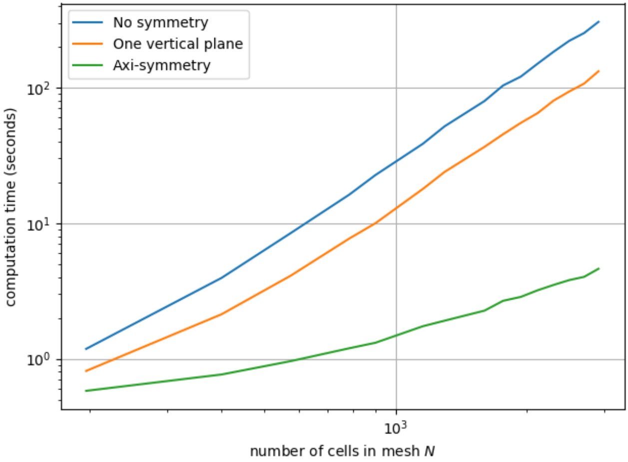




- Computation of $\lceil n/2 \rceil m^2$ coefficients instead of $(nm)^2$.
- Faster resolution of the linear system:
  - Block-diagonalisation of block circulant matrices with FFT $\Rightarrow$ n systems of size $m \times m$.

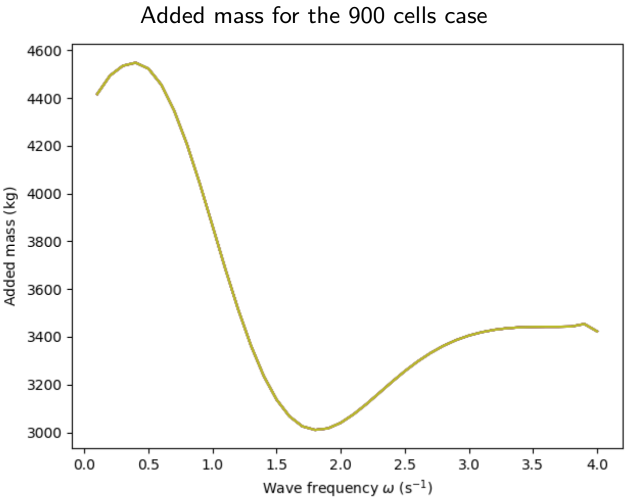
Benchmark

Comparison of the computation time



Benchmark

Comparison of the results

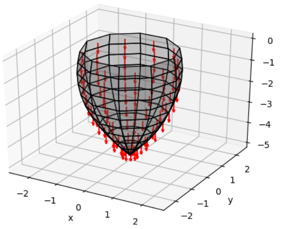


Relative error $\sim 10^{-4}$ (single precision rounding errors?)

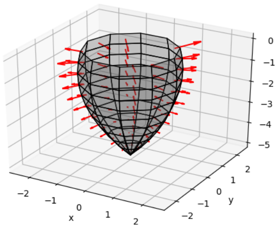
Not a symmetric problem

only a symmetric floating body

Axisymmetric problem
(Heave)

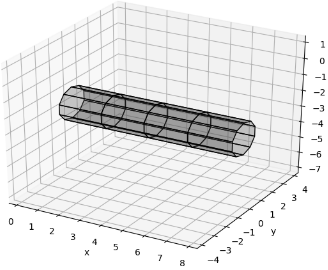


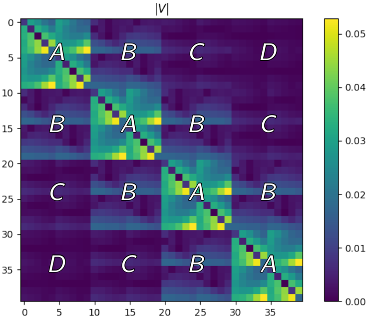
Non axisymmetric problem
(Elastic deformation)



Prismatic shape

Block symmetric Toeplitz structure





- Computation of nm^2 coefficients instead of $(nm)^2$.
- Faster resolution of the linear system?

Nested symmetries

```
half_ring = load_mesh_file("half_ring.dat")

ring = ReflectionSymmetry(half_ring,
                          xOz_Plane)

cylinder = TranslationalSymmetry(ring,
                                 x_direction,
                                 times=n)
```

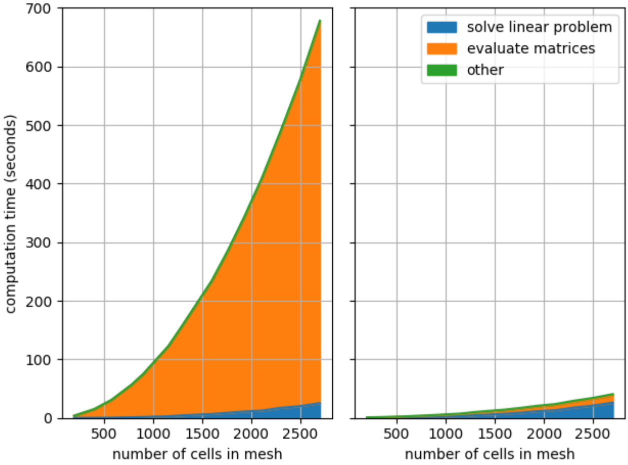
```
buoy = load_mesh_file("buoy.dat")

arm = TranslationalSymmetry(buoy,
                            x_direction,
                            times=10)
arm.rotate_z(pi/8)

weptos = ReflectionSymmetry(arm,
                            xOz_Plane)
```


SBM S3 (model scale)


Weptos

Benchmark

Finite depth



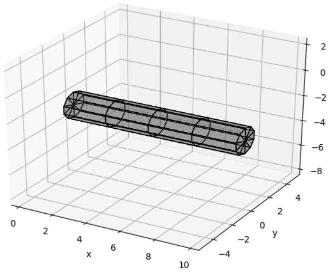
Conclusion

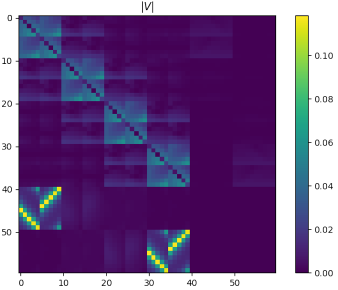
- Improvement of the efficiency of Nemoh
  - Using the symmetries of the floating body (= the domain);
  - Speed up $\sim \sqrt{N}$, where N is the number of cells;
  - For the same precision.
  - Also possibly less memory consumption (not studied here).
- Among other optimizations of Nemoh.
- Perspective: speed/accuracy trade-off?

Perspectives

Nearly symmetric bodies

Closed horizontal cylinder:

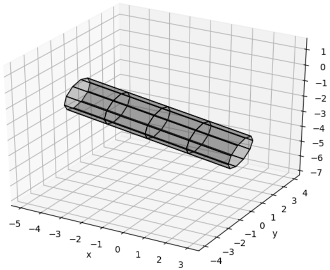


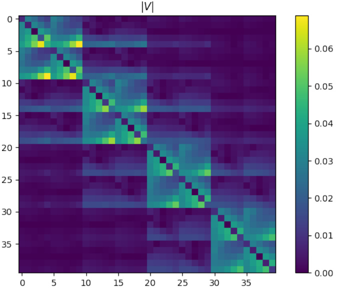


Perspectives

Nearly symmetric bodies

Tilted horizontal cylinder:

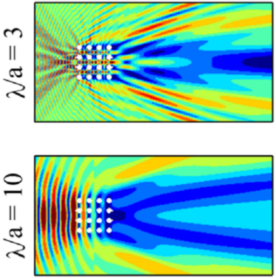




Perspectives
Arrays of WECs

Simulation of large regular arrays of identical WECs.

To be compared with other strategies such as *McNatt et al., 2014* and *Fabregas Flavia et al., 2018*.



McNatt et al., 2014

Thank you for your attention!

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