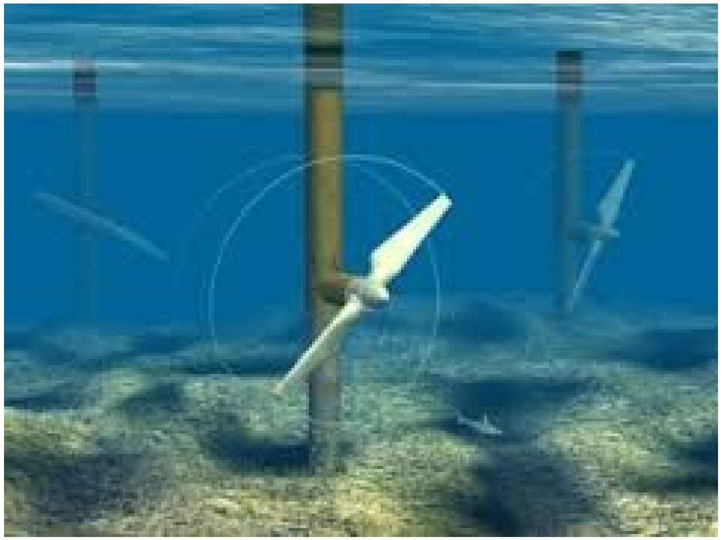
Vers une première validation du model SDM pour écoulements marins

On the first validations of the SDM model for tidal flows

Cyril Mokrani, Mireille Bossy, Antoine Rousseau

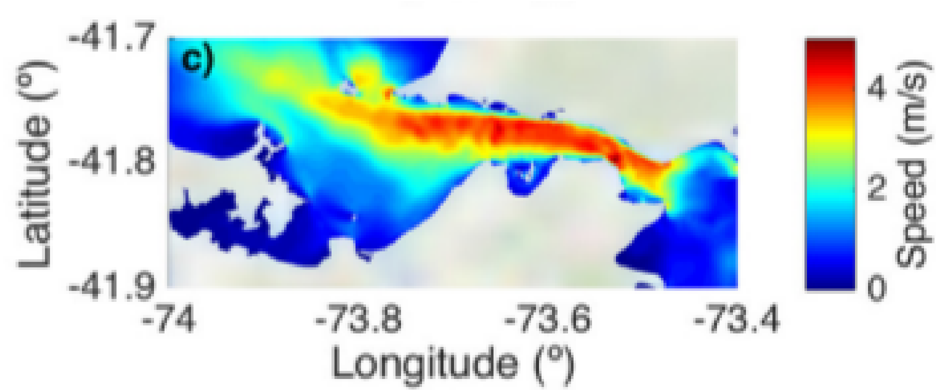
CONTEXT



→ Ocean/river energy extraction process

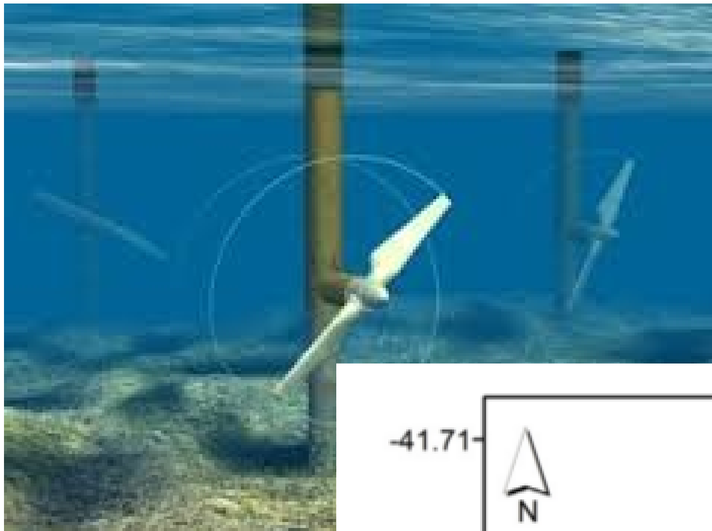
→ Starting project for Chilean

→ Example of application : “Canal Chacao”



Guerra et al. 2017

CONTEXT



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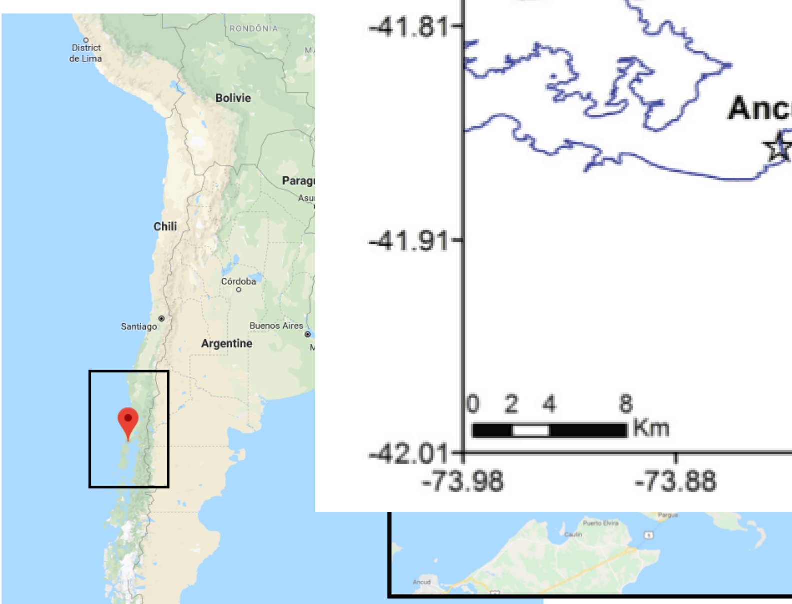
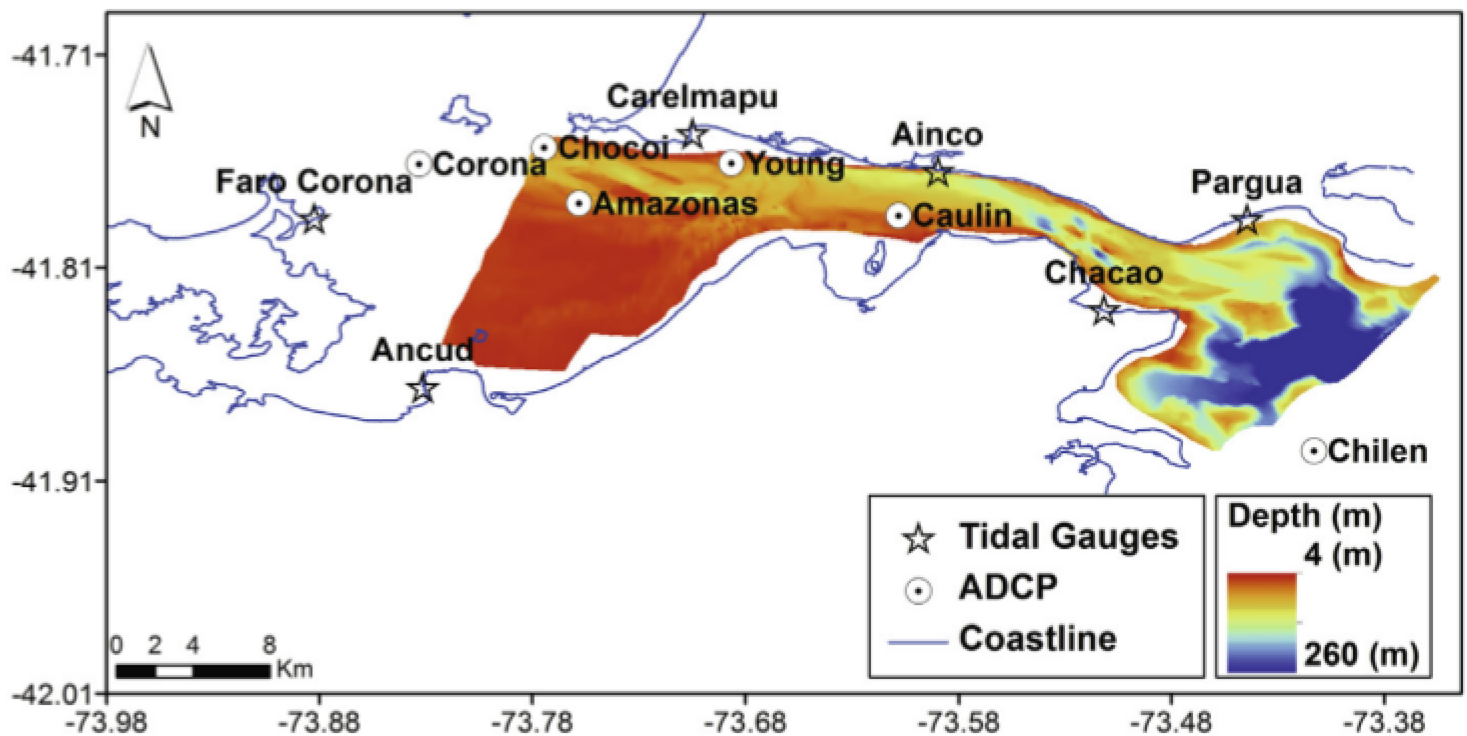
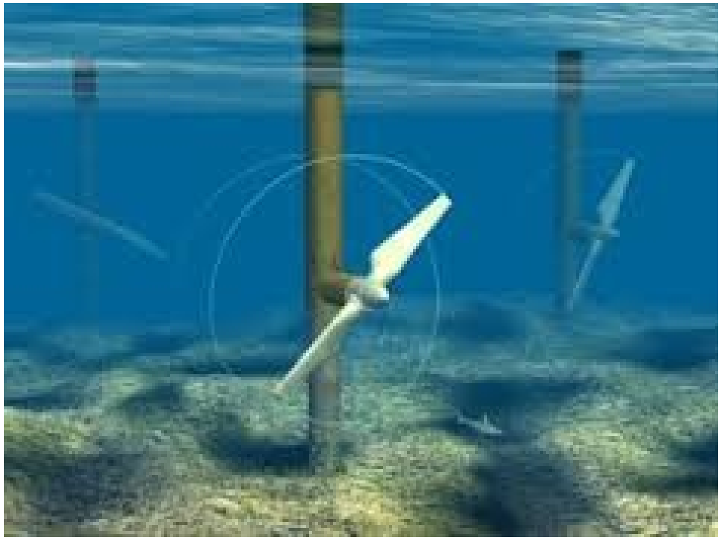




CONTEXT



→ Ocean/river energy extraction process

→ Starting project for Chilean

→ example of application : “Canal Chacao”

→ Numerical model : prediction of turbulence patterns

→ Existing version for wind turbines “SDM-windpos”

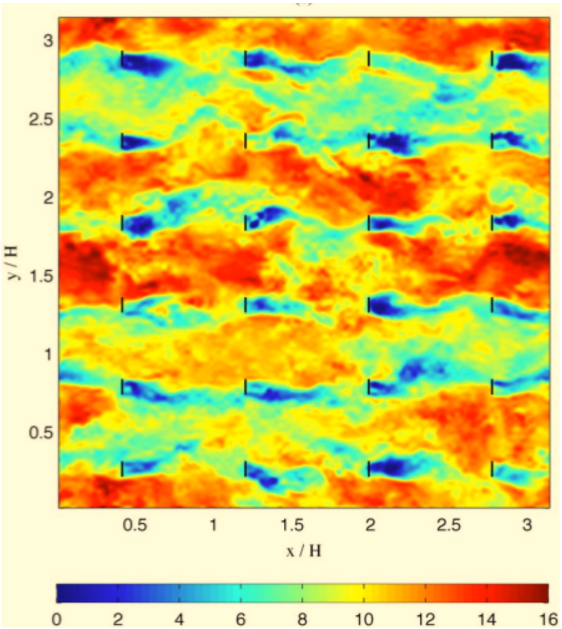
Can we apply it to ocean flows ?

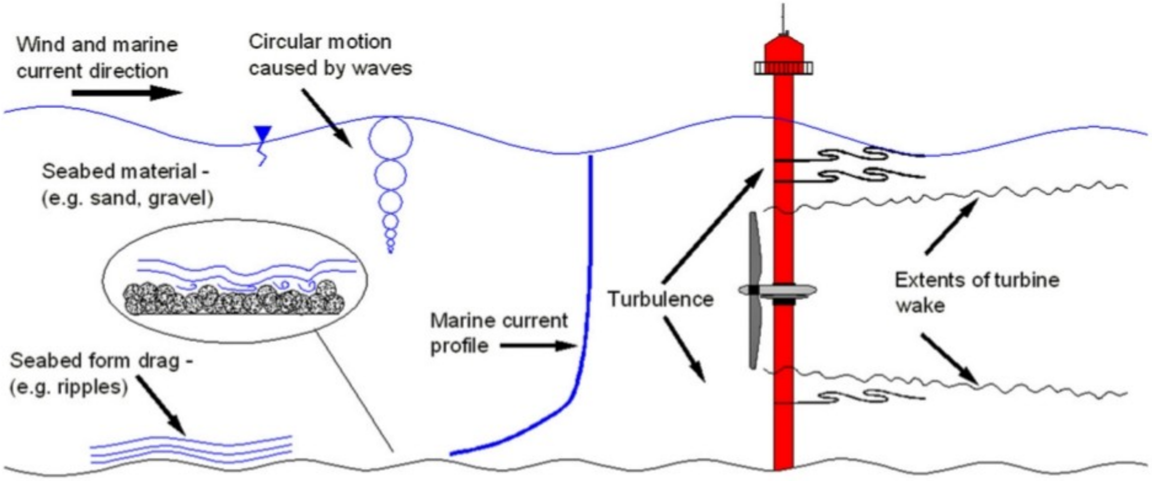
OBJECTIVES

→ Describe fluid dynamics upstream the turbine

→ Model the turbine effects on the fluids dynamics

→ Assess the numerical accuracy and performances





OBJECTIVES

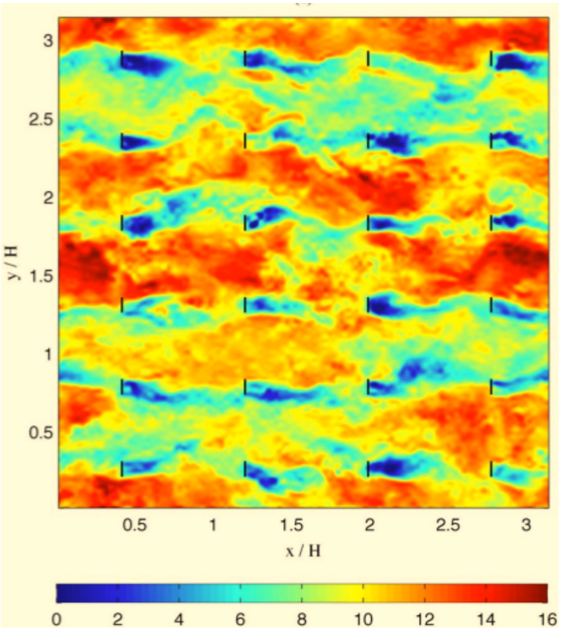
→ Describe fluid dynamics upstream the turbine

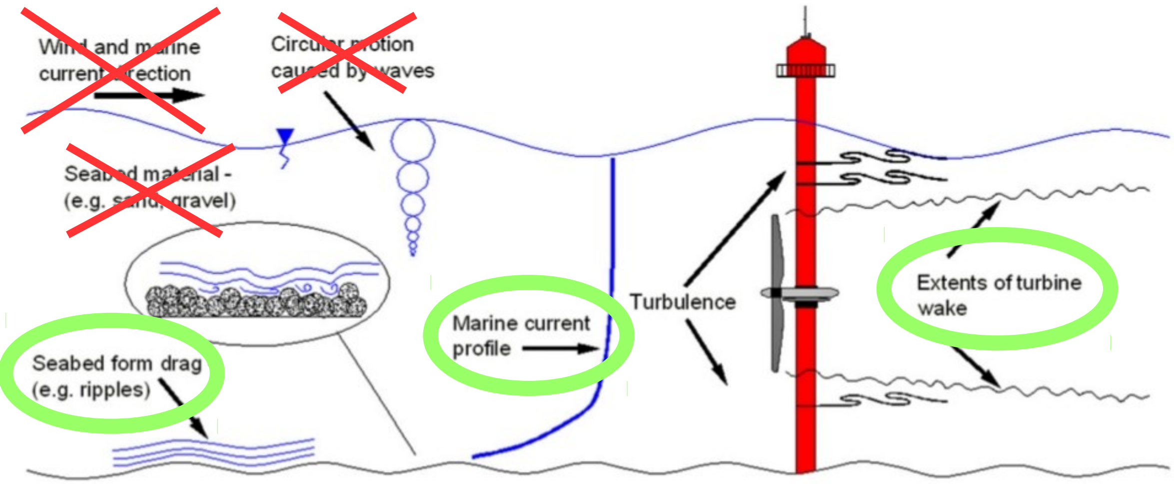
OB1. Boundary layers
OB2. Effect of bathymetry

→ Model the turbine effects on the fluids dynamics

OB3. Implementation of “simplified” turbine models

→ Assess the numerical accuracy and performances





CONTENTS

I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

$$
\begin{aligned}
dX_t &= \mathcal{U}_t dt \qquad \mathcal{U}_t = (\mathrm{u}_t^{(i)}, i = 1, 2, 3) \\
d\mathrm{u}_t^{(i)} &= -\partial_{x_i} \langle \mathscr{P} \rangle (t, X_t) dt + \left(\sum_j G_{ij} \left(\mathrm{u}^{(j)} - \langle \mathrm{u}^{(j)} \rangle \right) \right) (t, X_t) dt + \sqrt{C_0 \varepsilon (t, X_t)} dB_t^{(i)}.
\end{aligned}
$$

I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

Brownian motion

$$dX_t = \mathcal{U}_t\, dt \qquad \mathcal{U}_t = (\mathrm{u}_t^{(i)}, i = 1,2,3)$$

$$d\mathrm{u}_t^{(i)} = \boxed{-\partial_{x_i}\langle \mathscr{P}\rangle(t, X_t)dt + \left(\sum_j G_{ij}\left(\mathrm{u}^{(j)} - \langle \mathrm{u}^{(j)}\rangle\right)\right)(t, X_t)dt} + \boxed{\sqrt{C_0 \varepsilon(t, X_t)}dB_t^{(i)}}$$

Low accelerations terms

High accelerations terms

I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

Brownian motion

$$dX_t = \mathcal{U}_t dt \qquad \mathcal{U}_t = (\mathrm{u}_t^{(i)}, i = 1,2,3)$$

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Low accelerations terms — High accelerations terms

$\partial_{x_i}\langle\mathscr{P}\rangle(t,X_t)dt$ → Pressure gradient term [reduced pressure]

$\sum_j G_{ij}\left(\mathrm{u}^{(j)} - \langle\mathrm{u}^{(j)}\rangle\right)$ → “Drifter” term – Come back to the mean velocity

$\sqrt{C_0\varepsilon(t,X_t)}dB_t^{(i)}$ → “Stochastic diffusion” term – Driven by pseudo dissipation

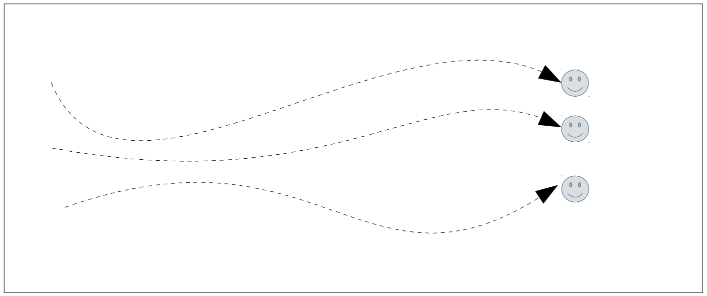
I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

$$d\mathrm{u}_t^{(i)} = -\partial_{x_i}\langle\mathscr{P}\rangle(t,X_t)dt + \left(\sum_j G_{ij}\left(\mathrm{u}^{(j)} - \langle\mathrm{u}^{(j)}\rangle\right)\right)(t,X_t)dt + \sqrt{C_0\varepsilon(t,X_t)}dB_t^{(i)}.$$

Mean velocity
In the particle cell



Numerical domain

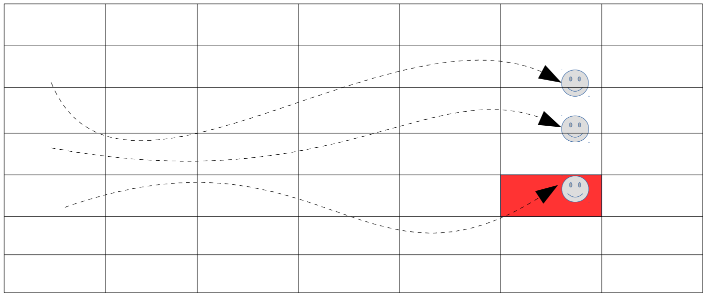
I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

$$d\mathrm{u}_t^{(i)} = -\partial_{x_i}\langle\mathscr{P}\rangle(t,X_t)dt + \left(\sum_j G_{ij}\left(\mathrm{u}^{(j)} - \langle\mathrm{u}^{(j)}\rangle\right)\right)(t,X_t)dt + \sqrt{C_0\varepsilon(t,X_t)}dB_t^{(i)}.$$

Mean velocity
In the particle cell



Eulerian mesh grid

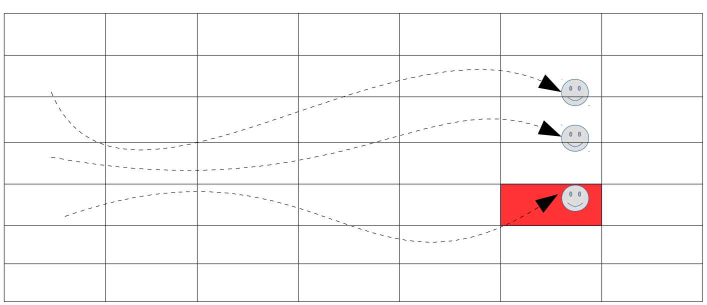
I. Numerical methods

- Lagrangian model based on PDF equation

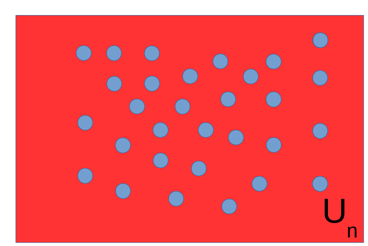
→ Incompressible flow

$$du_t^{(i)} = -\partial_{x_i}\langle \mathscr{P}\rangle(t,X_t)dt + \left(\sum_j G_{ij}\left(u^{(j)} - \langle u^{(j)}\rangle\right)\right)(t,X_t)dt + \sqrt{C_0\varepsilon(t,X_t)}\,dB_t^{(i)}.$$

Mean velocity
In the particle cell



Eulerian mesh grid



N = Number of particle /cell keeps constant

$< U > = \text{mean}(U_n)$, for n in [1,N]

I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

3x3 Tensor

Parameter

$$d\mathrm{u}_t^{(i)} \quad = \quad -\partial_{x_i}\langle \mathscr{P}\rangle(t, X_t)dt + \left(\sum_j G_{ij}\left(\mathrm{u}^{(j)} - \langle \mathrm{u}^{(j)}\rangle\right)\right)(t, X_t)dt + \sqrt{C_0\varepsilon(t, X_t)}dB_t^{(i)}.$$

Mean velocity
In the particle cell

I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

$$d\mathrm{u}_t^{(i)} = -\partial_{x_i}\langle\mathscr{P}\rangle(t,X_t)dt + \left(\sum_j G_{ij}\left(\mathrm{u}^{(j)} - \langle\mathrm{u}^{(j)}\rangle\right)\right)(t,X_t)dt + \sqrt{C_0\varepsilon(t,X_t)}dB_t^{(i)}.$$

→ IP model :

$$C_0 = \frac{2}{3}\left(C_R + C_2\frac{\mathcal{P}}{\varepsilon} - 1\right) \qquad G_{ij} = -\frac{C_R}{2}\frac{\varepsilon}{k}\delta_{ij} + C_2\frac{\partial\langle\mathrm{u}^{(i)}\rangle}{\partial x_j}$$

$$C_R = 1.8$$

$$C_2 = 0.6\min\left\{1, C_\nu\frac{\det\langle\mathrm{u}_i' u_j'\rangle}{(\frac{2}{3}k)^3}\right\} \qquad C_\nu = 3.4$$

$$\mathcal{P} = \frac{1}{2}\left(\mathcal{P}_{11} + \mathcal{P}_{22} + \mathcal{P}_{33}\right)$$

$$\mathcal{P}_{ij} := -\sum_k\left(\langle\mathrm{u}^{(i)\prime}\mathrm{u}^{(k)\prime}\rangle\frac{\partial\langle\mathrm{u}^{(i)}\rangle}{\partial x_k} + \langle\mathrm{u}^{(j)\prime}\mathrm{u}^{(k)\prime}\rangle\frac{\partial\langle\mathrm{u}^{(j)}\rangle}{\partial x_k}\right)$$

I. Numerical methods

- Lagrangian model based on PDF equation

→ Incompressible flow

IP model

$$du_t^{(i)} = -\partial_{x_i}\langle \mathscr{P}\rangle(t, X_t)dt + \left(\sum_j G_{ij}\left(u^{(j)} - \langle u^{(j)}\rangle\right)\right)(t, X_t)dt + \sqrt{C_0\varepsilon(t, X_t)}dB_t^{(i)}.$$

Reduced pressure P/ρ

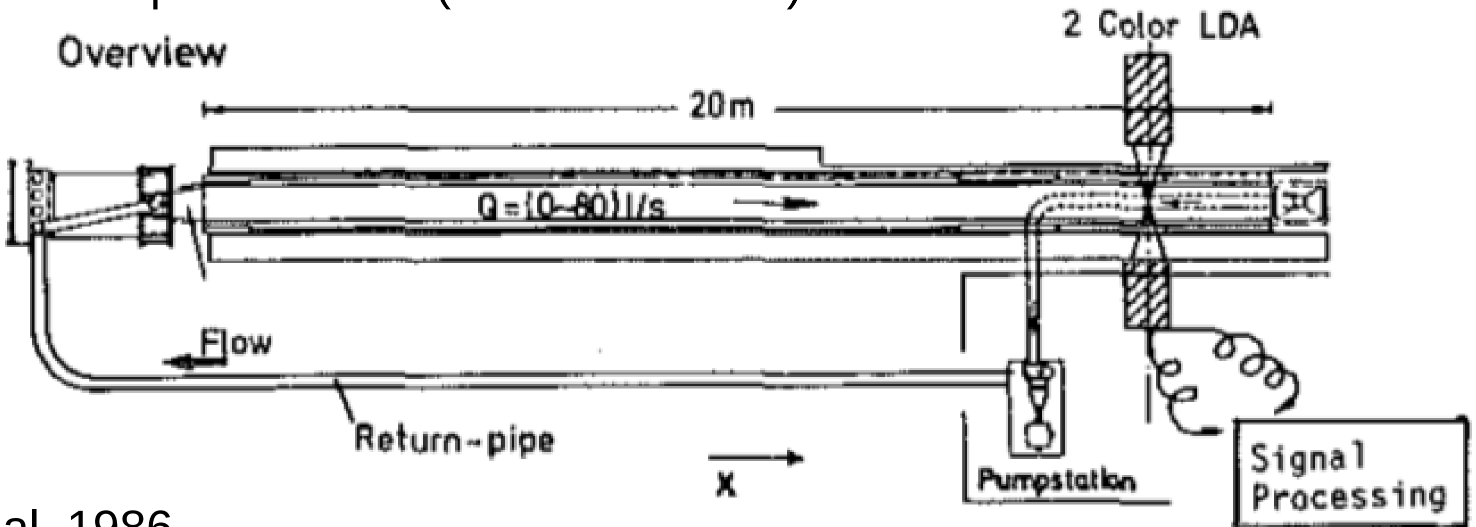
Mean velocity
In the particle cell

Brownian motion

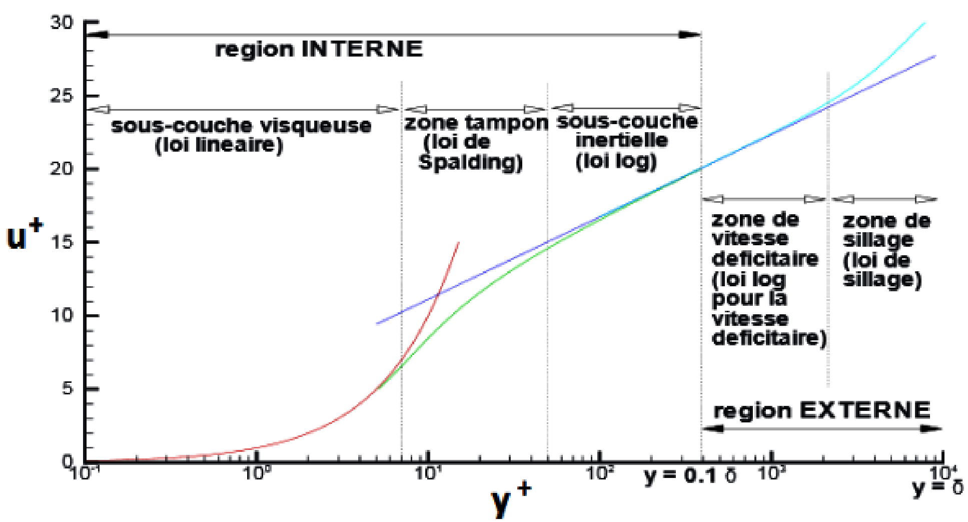
II. Benchmarks

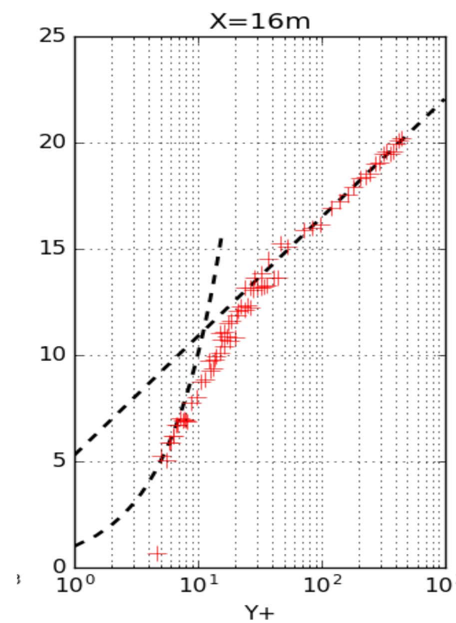
II.1 Description of the boundary layers

+ Benchmark on upstream flow (without turbines)



Nezu et al. 1986





II. Benchmarks

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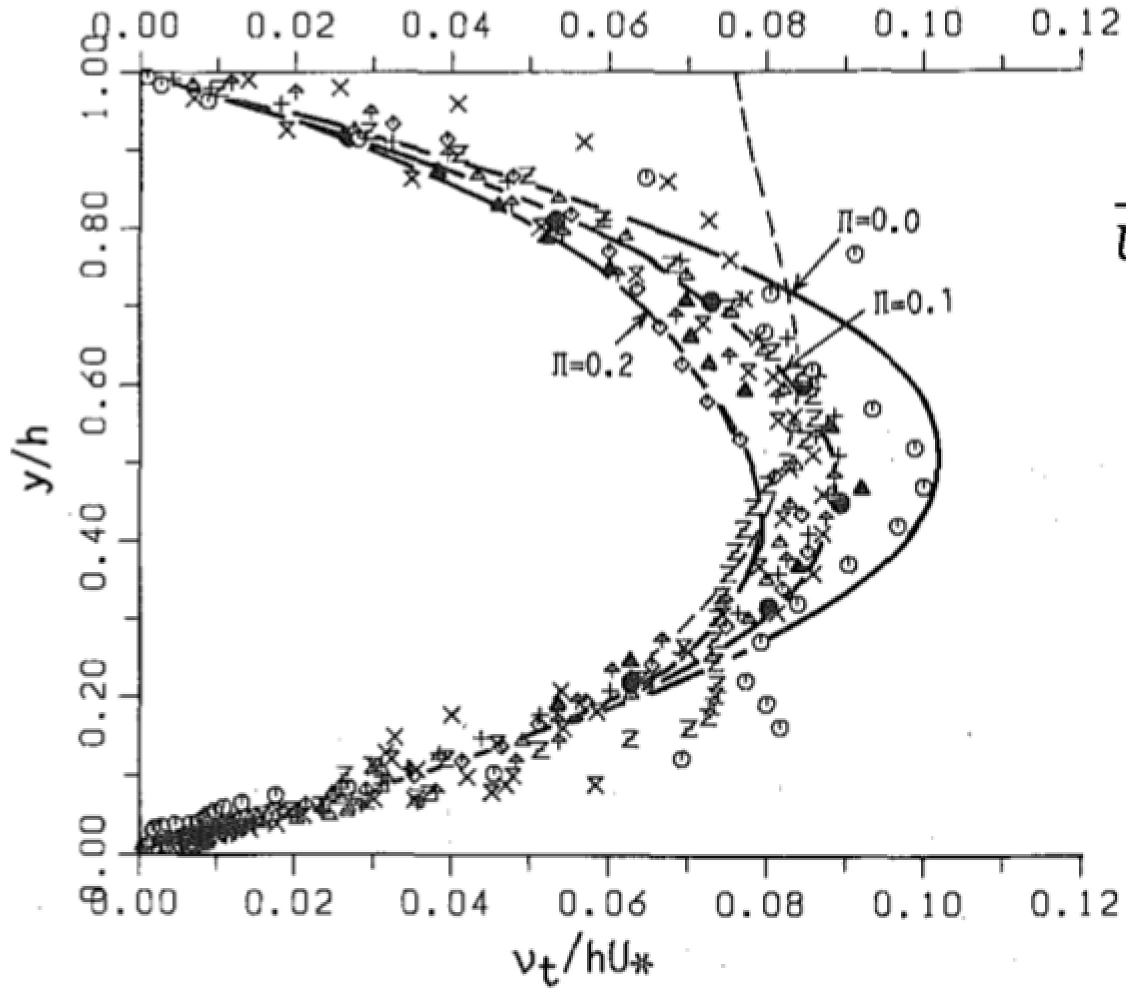
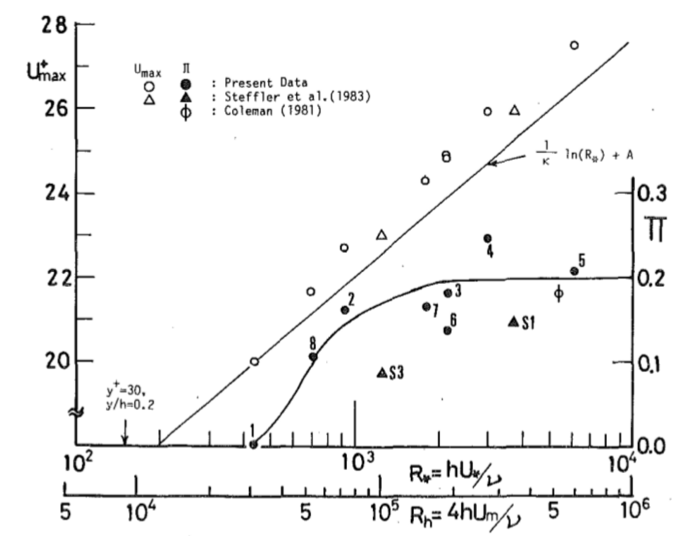


$$\frac{\nu_t}{U_* h} = \kappa(1-\xi)\left[\frac{1}{\xi} + \pi\Pi \sin(\pi\xi)\right]^{-1}$$



II. Benchmarks

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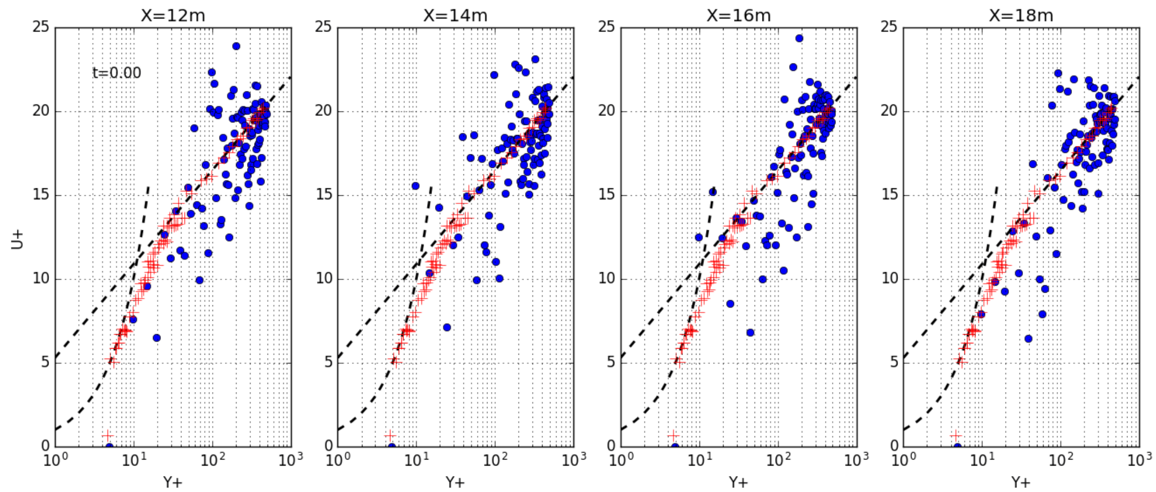
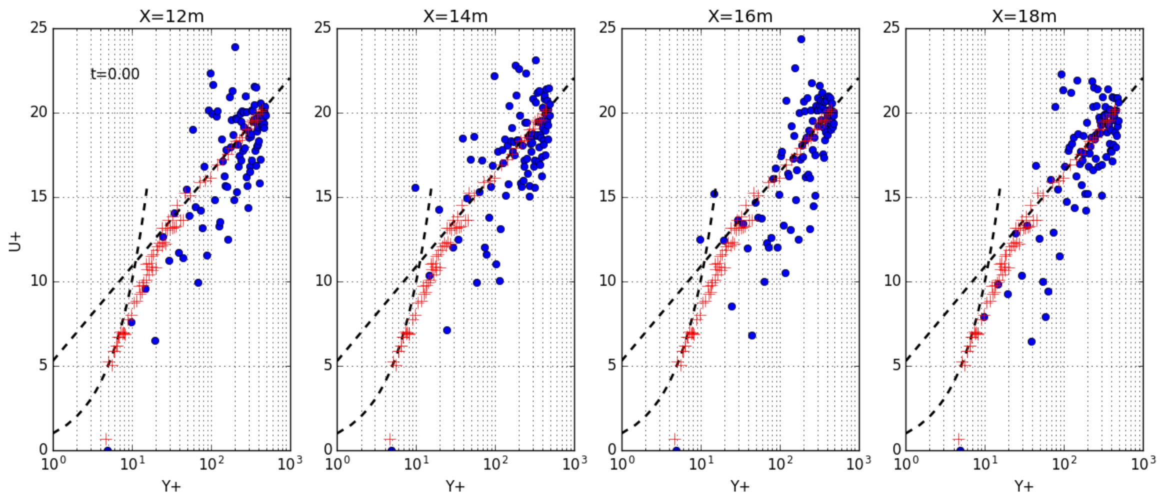
Old viscosity profile



New viscosity profile

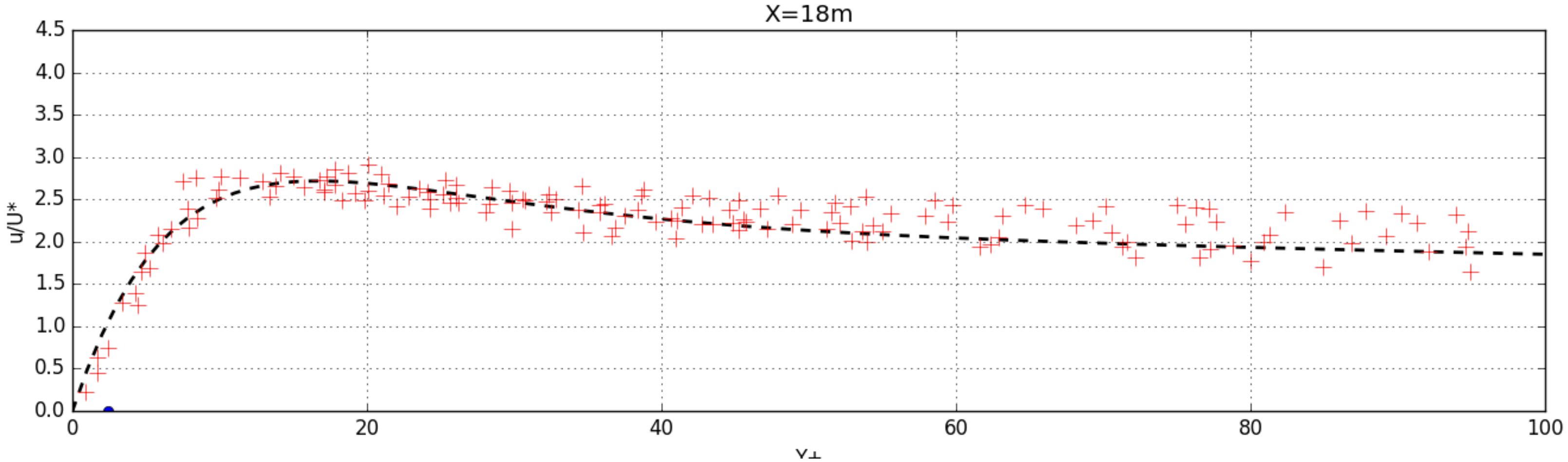


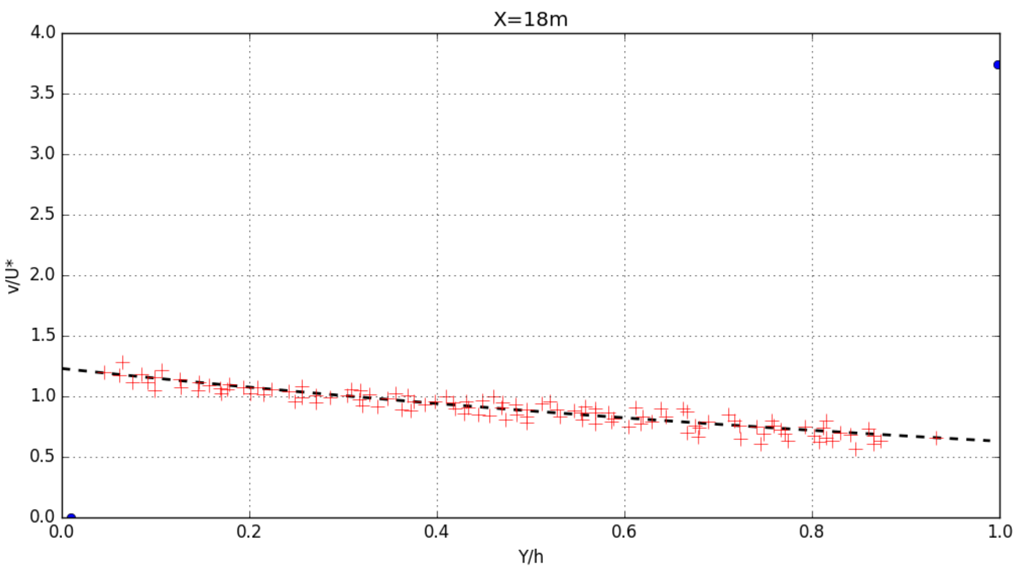
- - : Theoretical **+ +** : Experimental **o o** : Numerical

II. Benchmarks

II.1 Description of the boundary layers

+ Benchmark on upstream flow (without turbines)





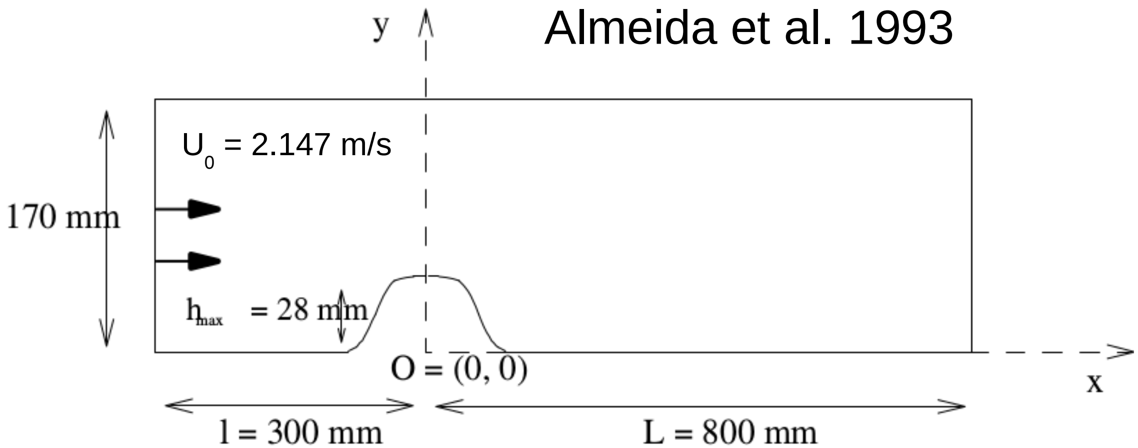
- - : Theoretical + + : Experimental o o : Numerical

II. Benchmarks

II.2 Effects of the bathymetry

+ Benchmark on upstream flow (without turbines)

Almeida et al. 1993



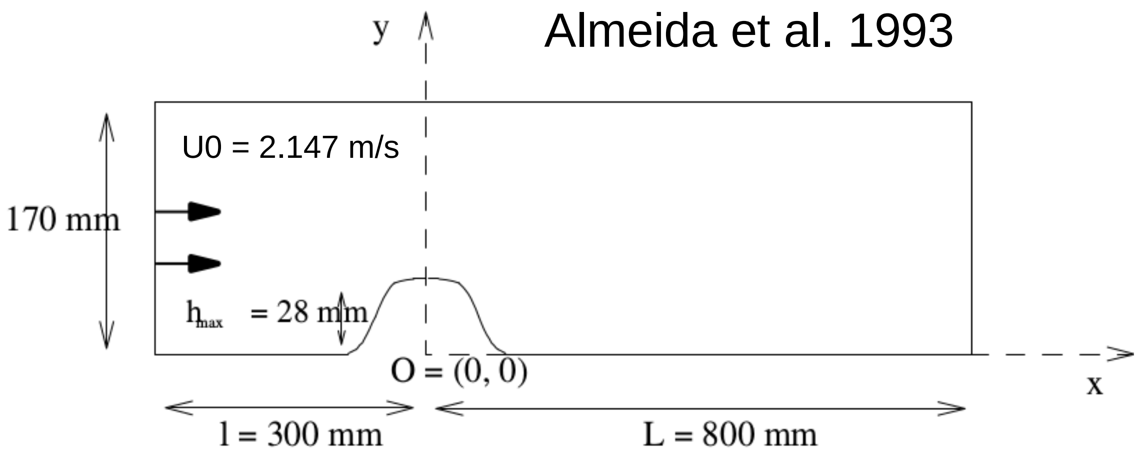
Re = 6.10^4

II. Benchmarks

II.2 Effects of the bathymetry

+ Benchmark on upstream flow (without turbines)

Almeida et al. 1993



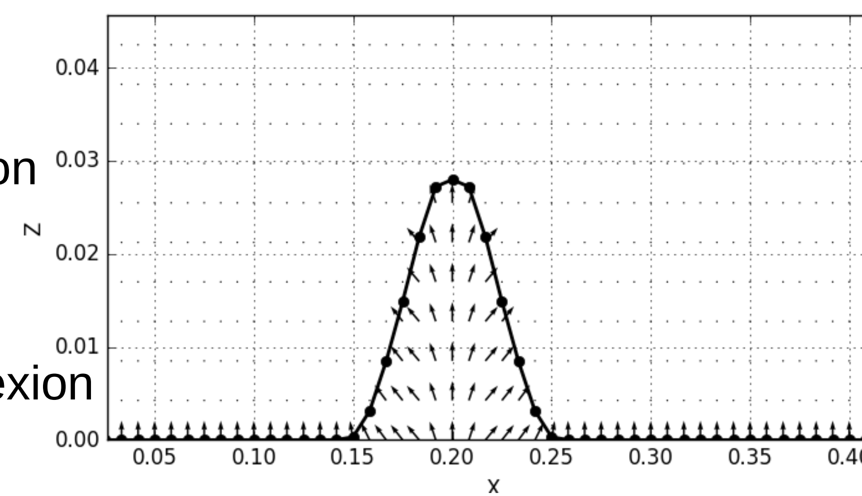
Re = 6.10^4

Numerical method :

- Reflexion of particles according to the local inclination

- On the ground, set the variances to get a log law

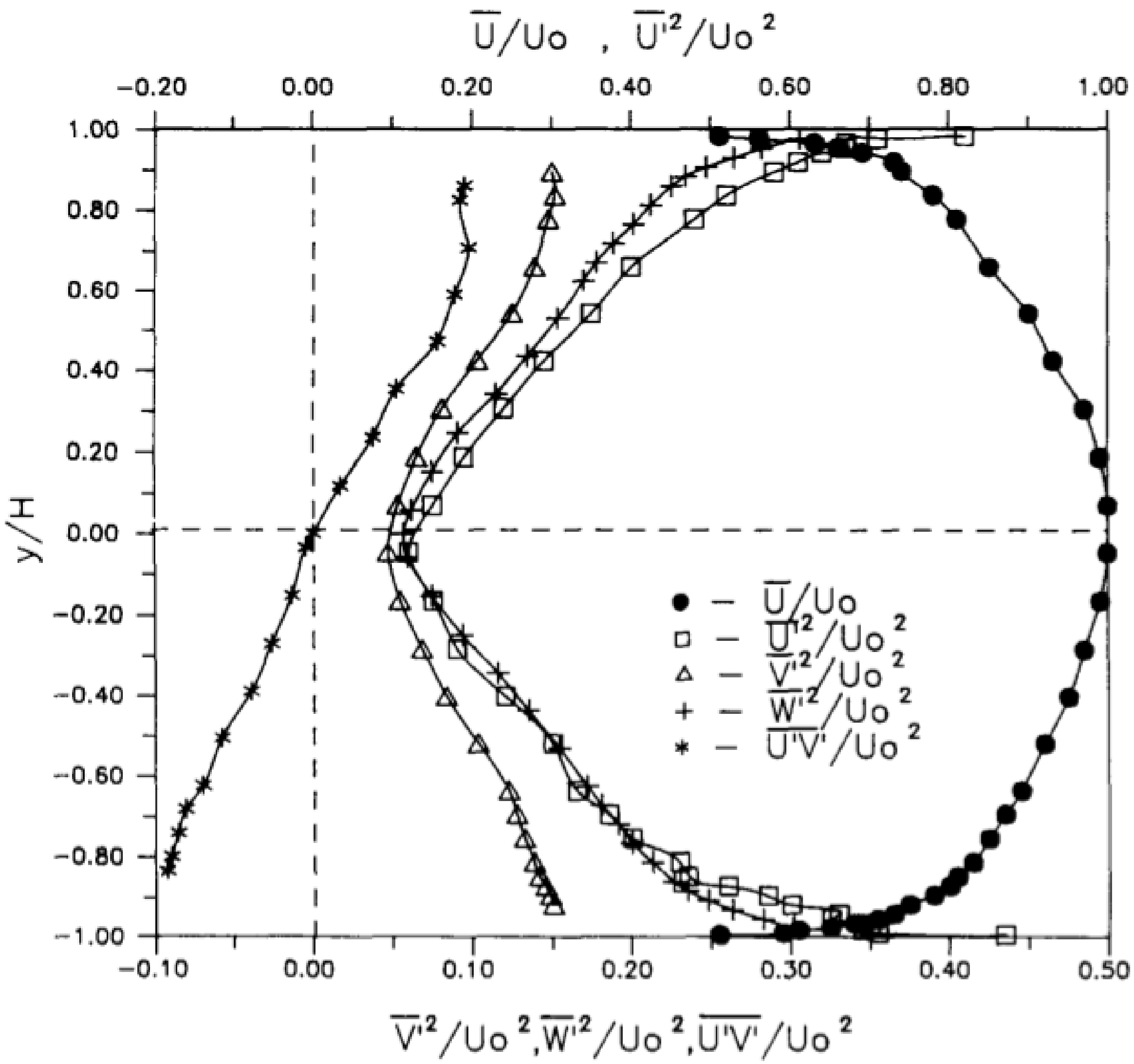
- Conservation of the covariance after/before the reflexion



II. Benchmarks

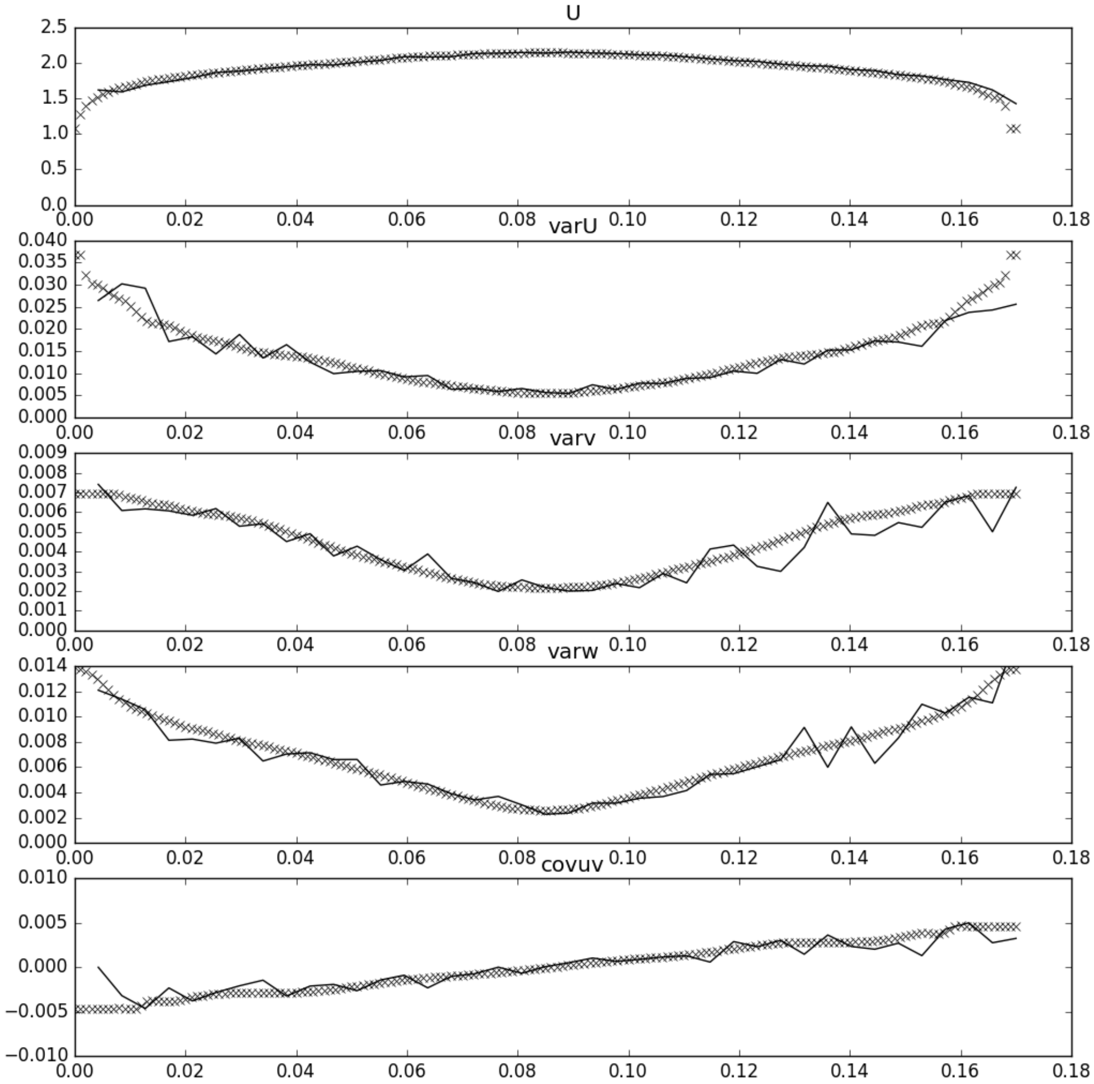
II.2 Effects of the bathymetry

+ First run without the hill



II. Benchmarks

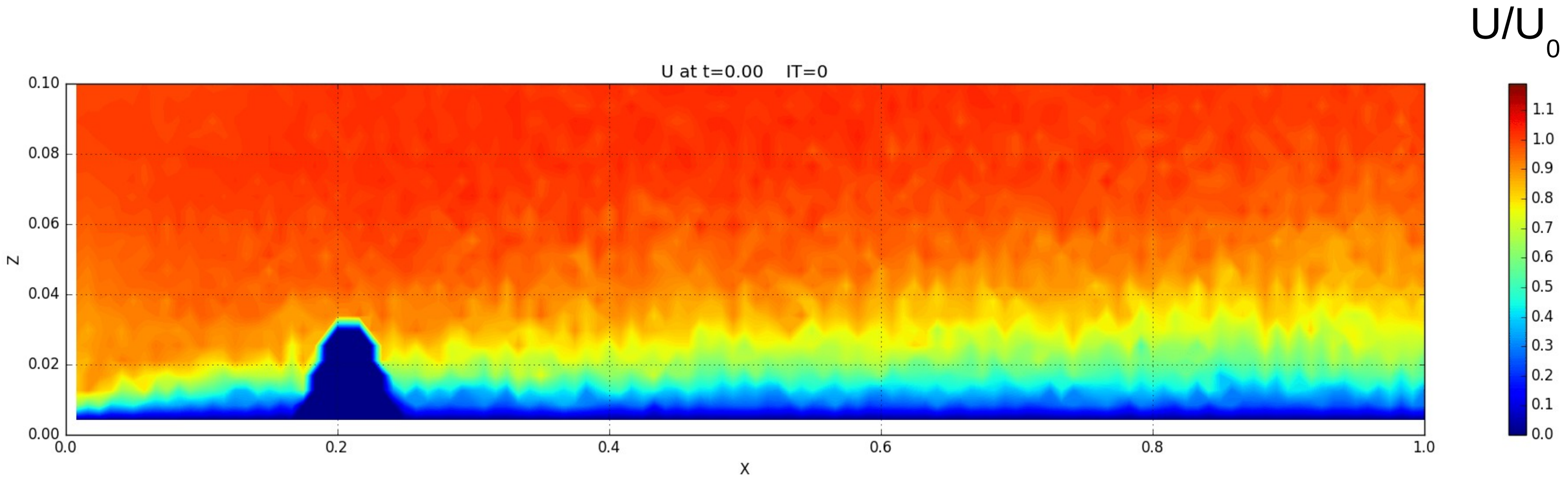
II.2 Effects of the bathymetry



II. Benchmarks

II.2 Effects of the bathymetry

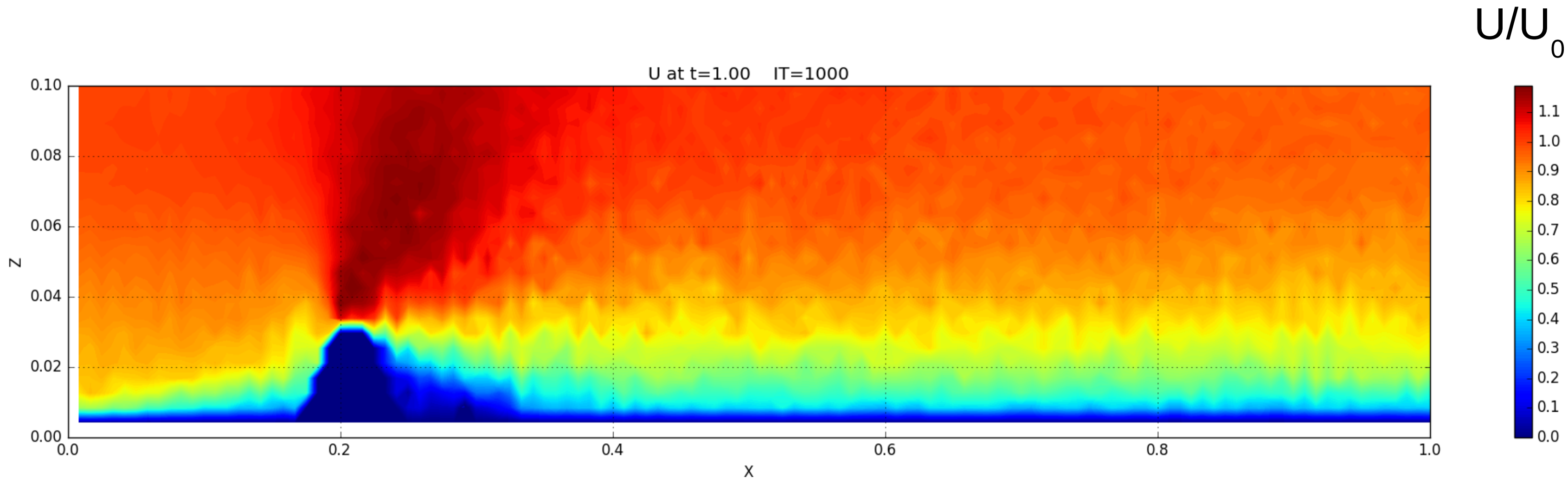
+ Restart the computation including the hill



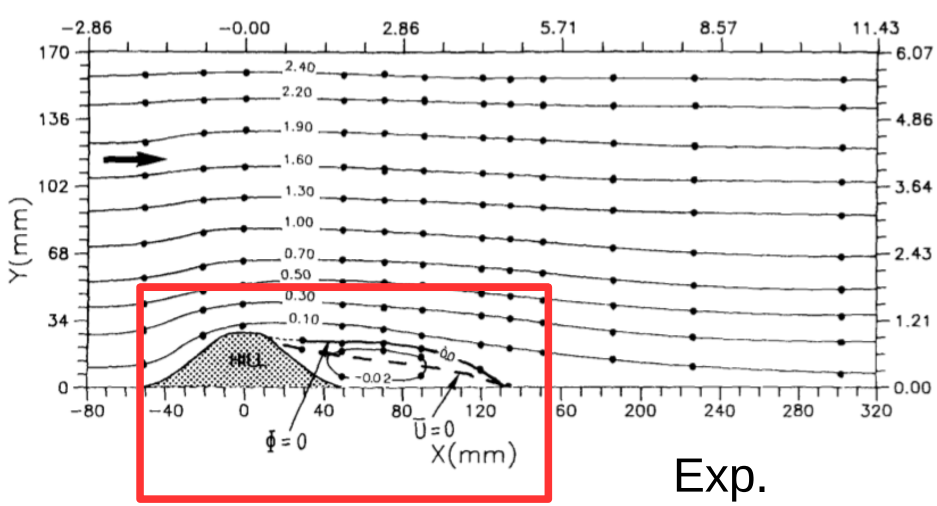
II. Benchmarks

II.2 Effects of the bathymetry

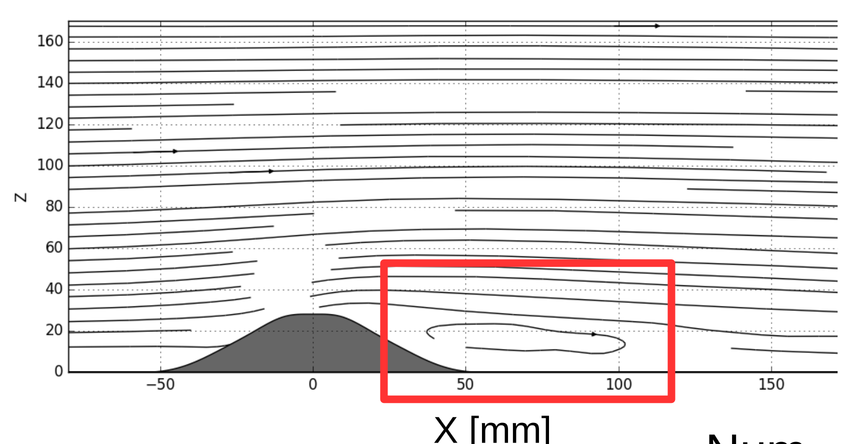
+ Mean velocity field



Field distribution of streamline



Exp.



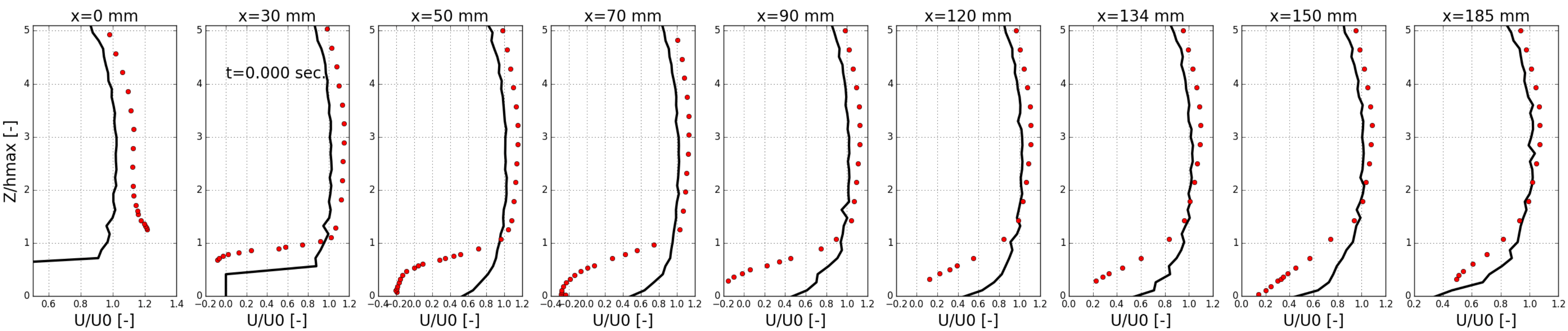
Num.

II. Benchmarks

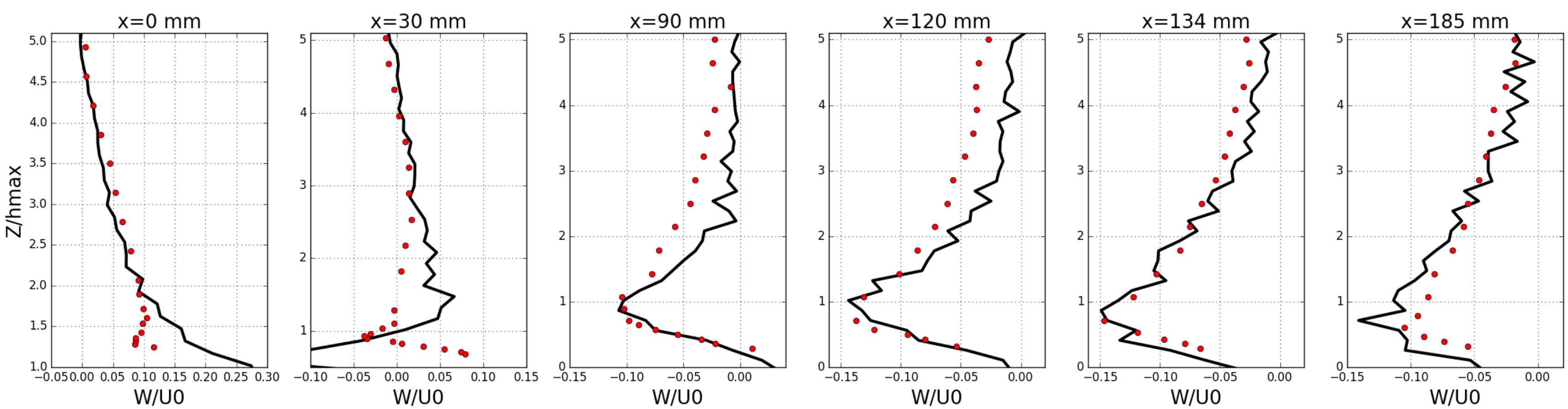
II.2 Effects of the bathymetry

U_0 = 2.147 m/s

+ Mean velocity profiles



~ h_{max}

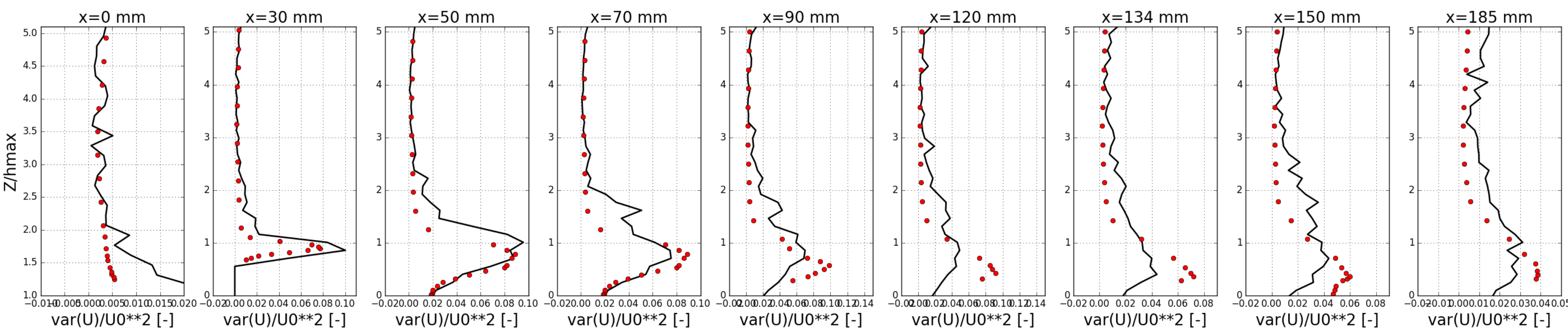


II. Benchmarks

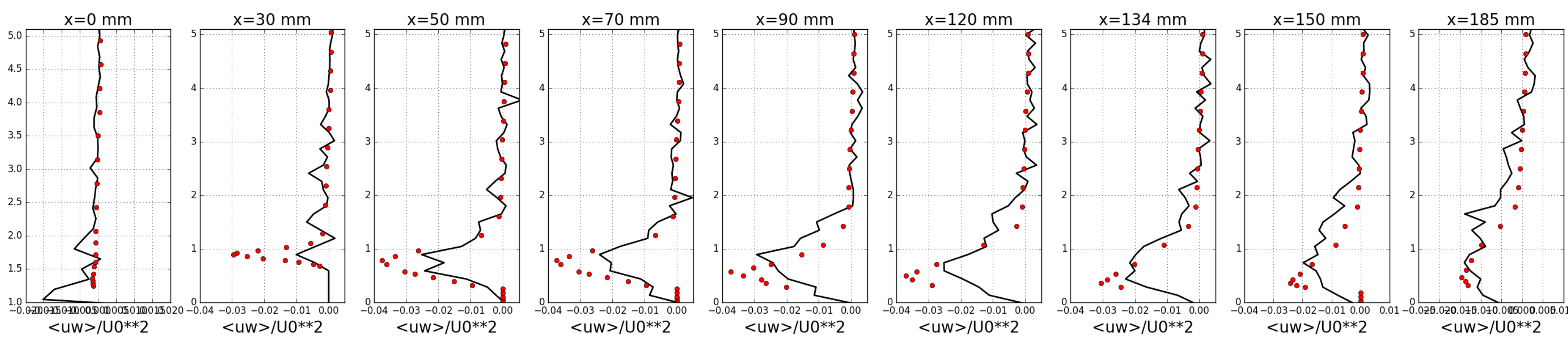
II.2 Effects of the bathymetry

U_0 = 2.147 m/s

+ Velocity fluctuations



+ Velocity correlations

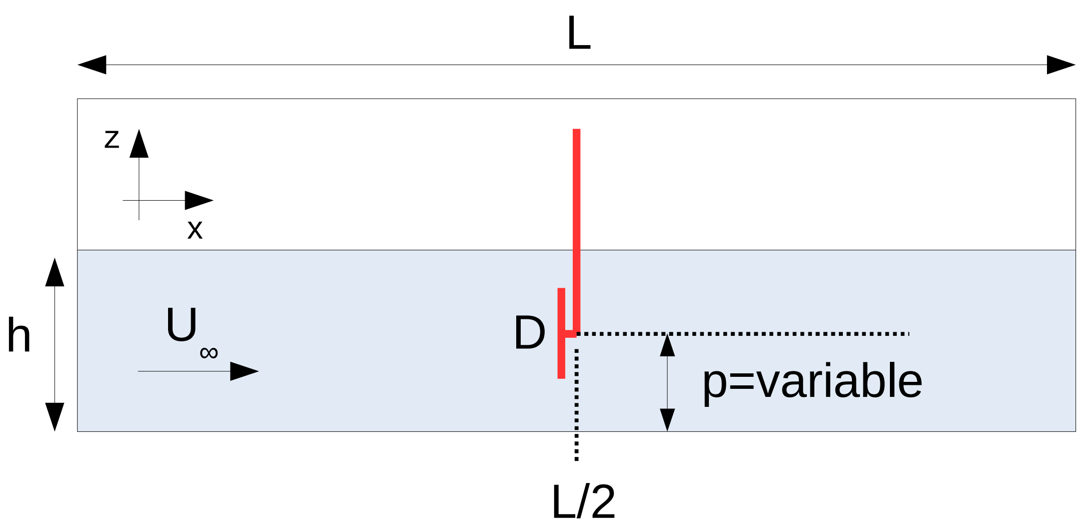


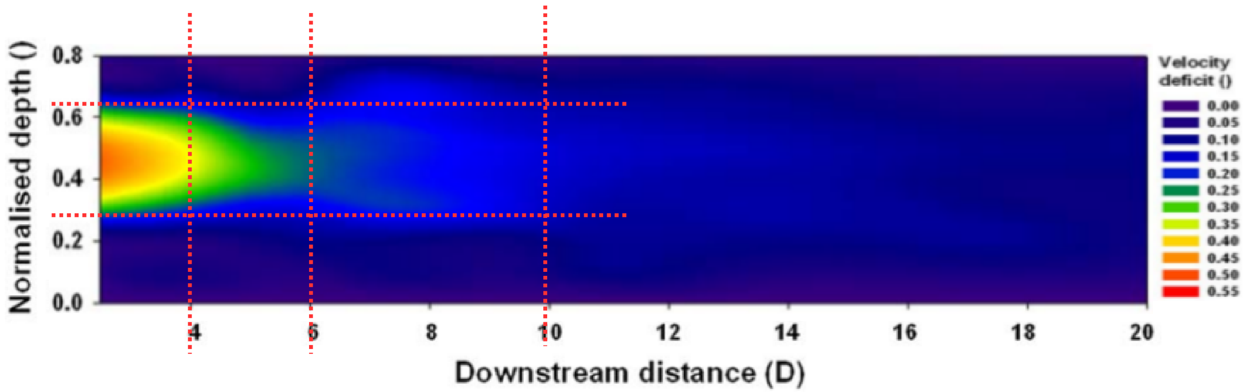
II. Benchmarks

II.3 Turbulence generated downstream turbines

Myers et al. 2010

Ct = variable





Turbulent length scales (x,z) :

X : 4D,6D,10D → high, medium, and low deficit

Z : Deficit over ~D with intermediate region over ~1/10 D

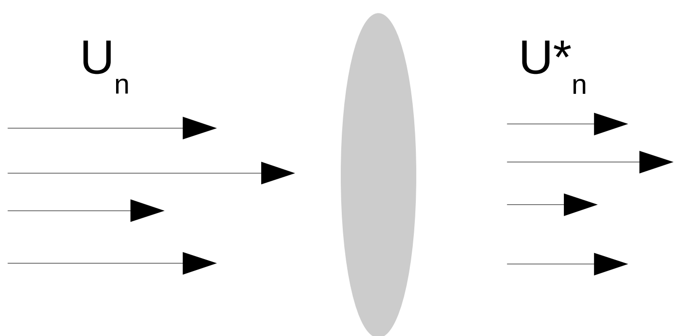
II. Benchmarks

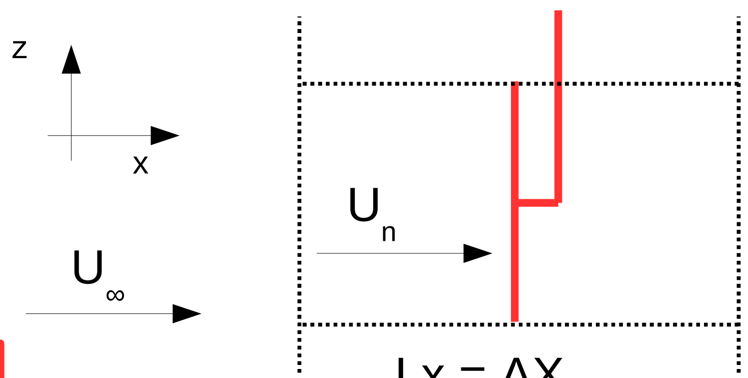
II.3 Turbulence generated downstream turbines

Model 1-D without rotation “porous disk”

First Model

$$F[N] = -\frac{1}{2}\rho A C_T U_\infty^2 \longrightarrow \Delta U = -\frac{1}{2} C_T U_\infty^2 \frac{\Delta t}{L_x}$$





Lx = ΔX
Cell length

$$C_T[-] = 4a(1-a) \qquad U_n = (1-a)U_\infty$$

$$U_n^* = U_n.\left(1 - \frac{2a}{1-a}\frac{\Delta t}{\Delta X}U_n\right)$$

II. Benchmarks

II.3 Turbulence generated downstream turbines

Model 1-D without rotation "porous disk"

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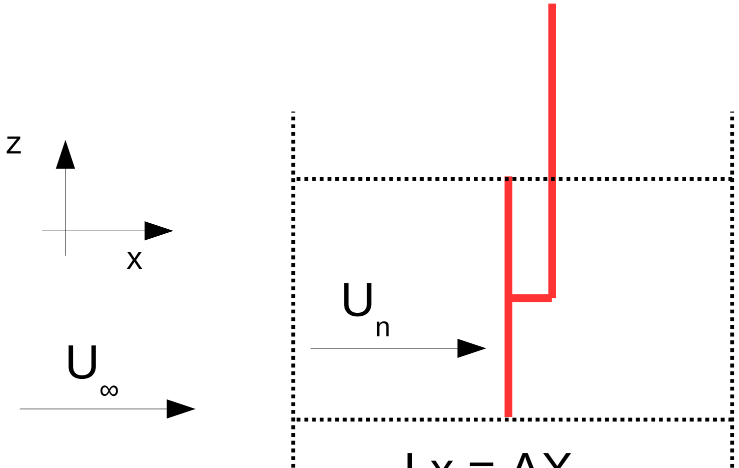
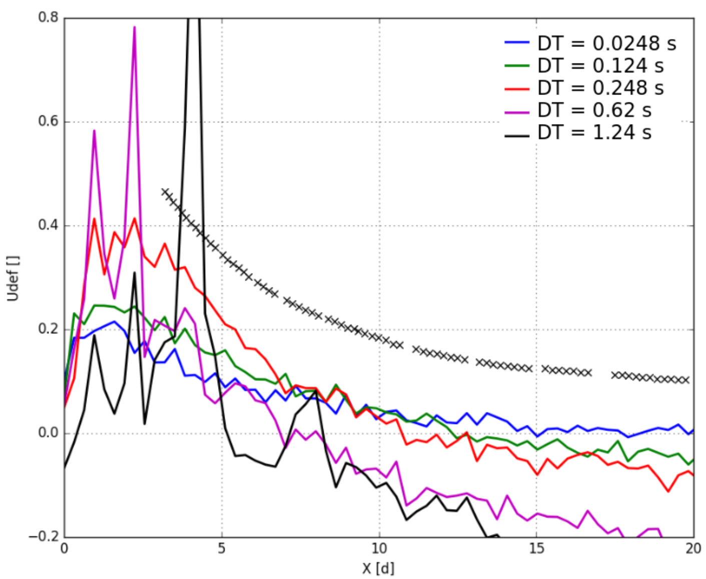




Dependence to time step :

- High DT → Better close to the disk
- Low DT → Better far from the disk

Difficult to configurate

II. Benchmarks

II.3 Turbulence generated downstream turbines

Model 1-D without rotation “porous disk”

Model including particle volume

$$F_{particle} = -\frac{1}{2}\rho A C_T U_\infty^2 \frac{\mathcal{V}_p}{\mathcal{V}_T}$$

Volume of 1 particle ($\mathcal{V}_p$)

Total Controle volume ($\mathcal{V}_T$)

$$\mathcal{V}_T = N_T \mathcal{V}_p$$

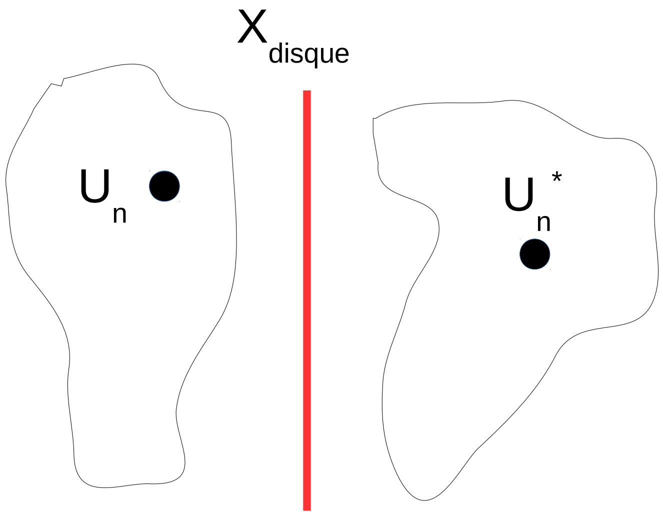
$$U_n^* = U_n - 2a(1-a)U_\infty^2 \frac{\Delta t}{\Delta X} \frac{A}{\Delta Y \Delta Z} \frac{N_{part/cell}}{N_T}$$

II. Benchmarks

II.3 Turbulence generated downstream turbines

Model 1-D without rotation "porous disk"

Model including particle volume



Vt = Volume of particles that cross the disk
between t_n and t_{n+1}

Vp = Volume of one particle
Volume of cell / Number of cell per cell

$$F_{particle} = -\frac{1}{2}\rho A C_T U_\infty^2 \frac{\mathcal{V}_p}{\mathcal{V}_T}$$

Volume of 1 particle ($\mathcal{V}_p$)

Total Controle volume ($\mathcal{V}_T$)

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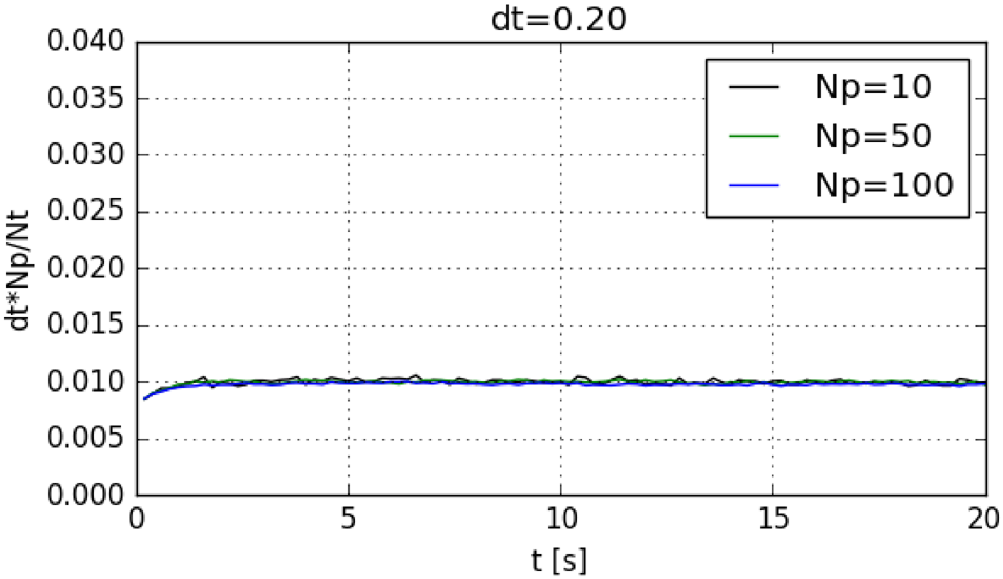
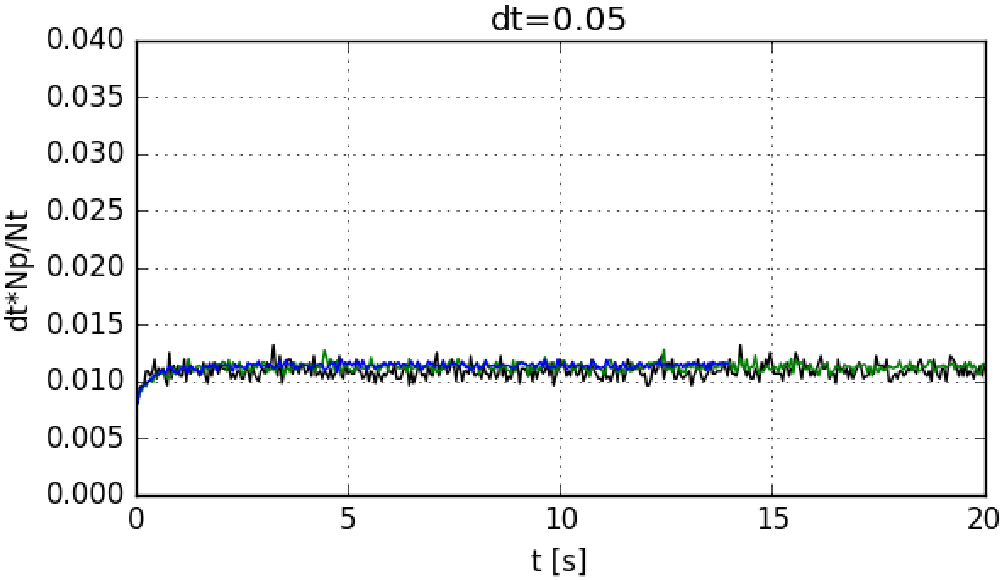
$$\mathcal{V}_T = N_T \mathcal{V}_p$$

$$U_n^* = U_n - 2a(1-a)U_\infty^2 \frac{\Delta t}{\Delta X} \frac{A}{\Delta Y \Delta Z} \frac{N_{part/cell}}{N_T}$$



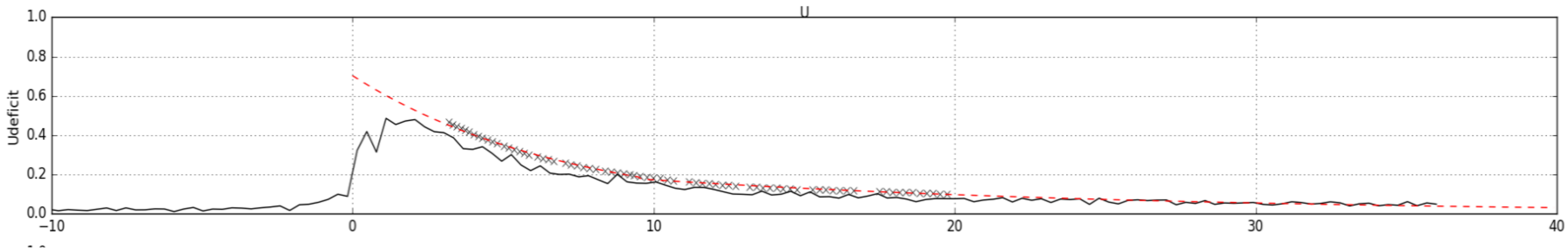


II. Benchmarks

II.3 Turbulence generated downstream turbines

Model 1-D without rotation "porous disk"

Model including particle volume



II. Benchmarks

II.3 Turbulence generated downstream turbines

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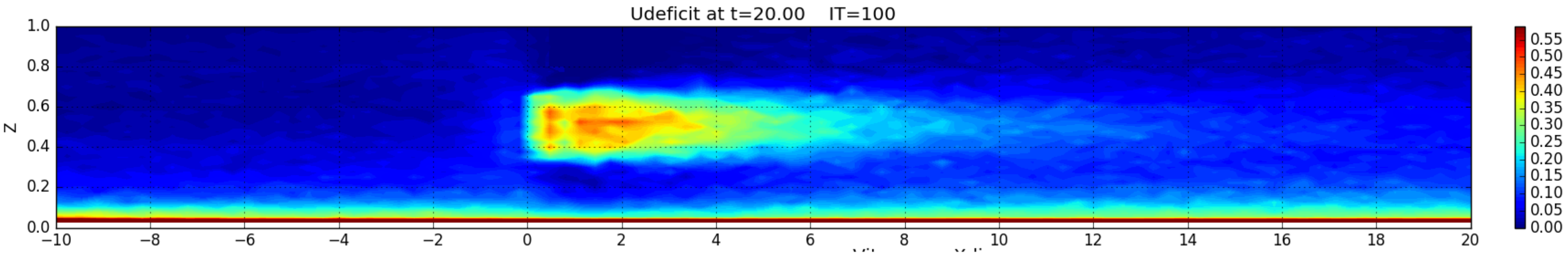
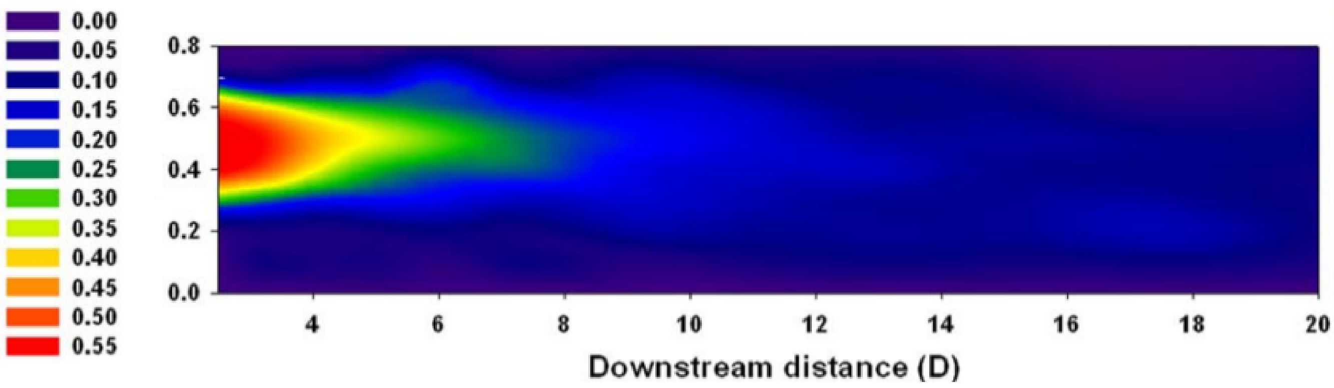




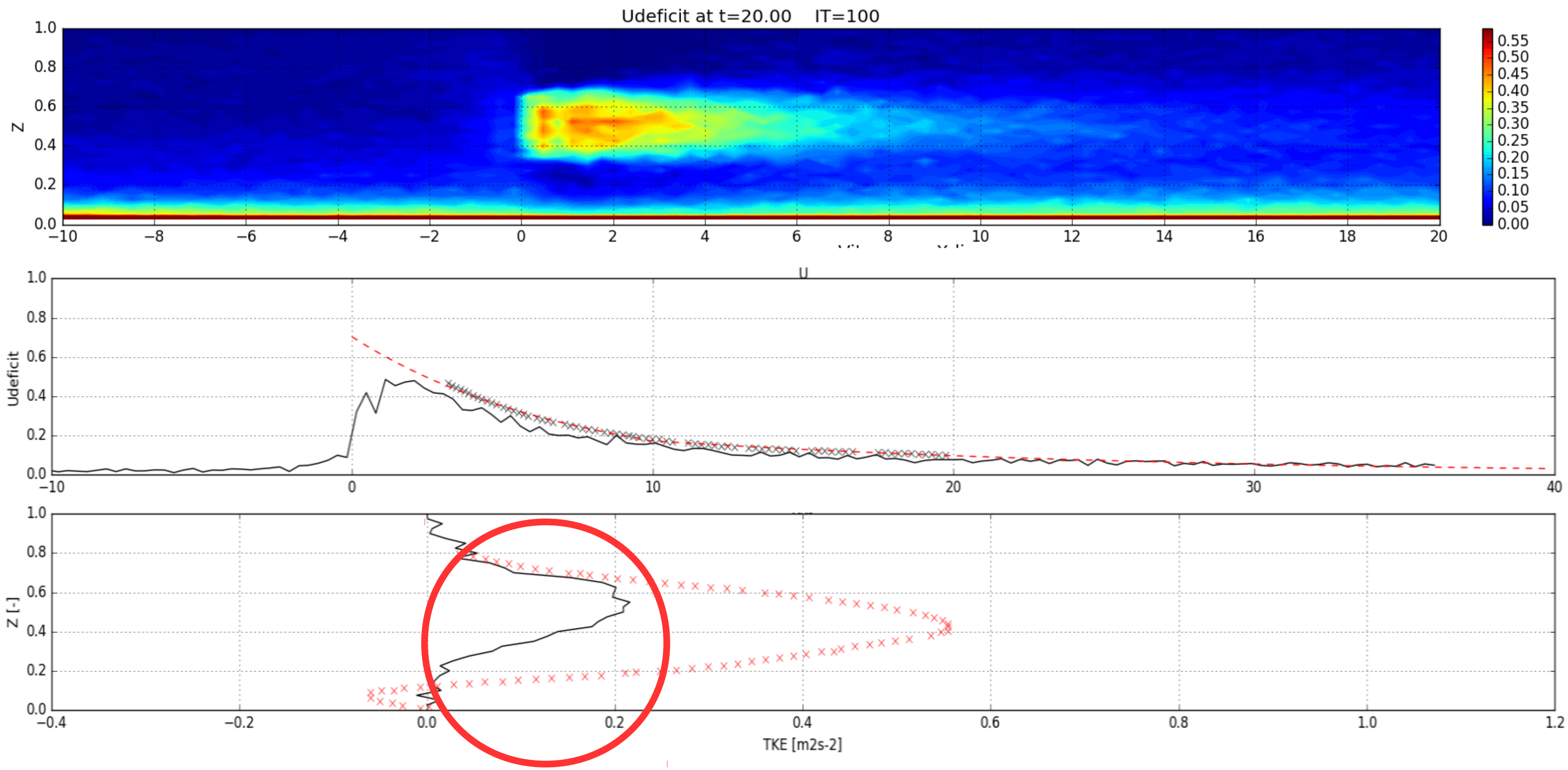
Repartition of Velocity deficit : no so good !

II. Benchmarks

II.3 Turbulence generated downstream turbines

Model 1-D without rotation “porous disk”

Model including particle volume



Significant under-estimation of TKE

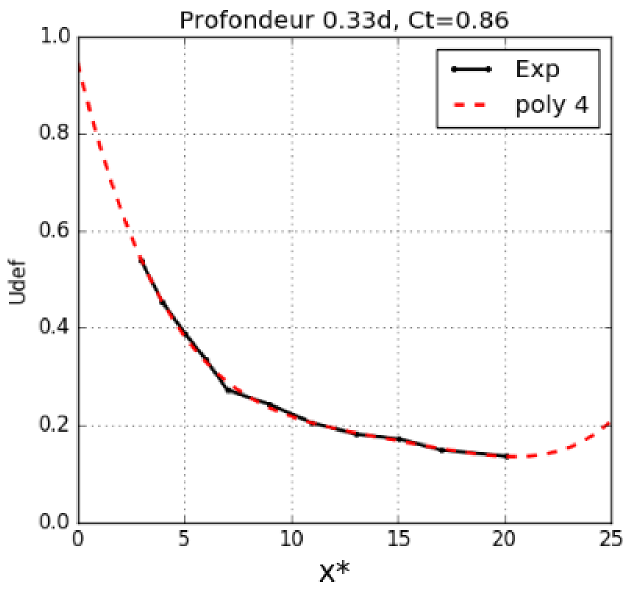
II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

→ New approach to get better flow statistics

$$G_{ij}(t, X_t)\,(U_t - < U >) \longrightarrow G_{ij}(t, X_t)\,(U_t - U_{ref})$$

$$U_{ref} = \frac{\alpha}{\alpha + \alpha'} < U > + \frac{\alpha'}{\alpha + \alpha'} U_{myers}(x^*, a)$$



$$\longrightarrow \quad U_{myers} = U_{\infty}(1 - U_{deficit} f(z^*))$$

Vertical ponderation

II. Benchmarks

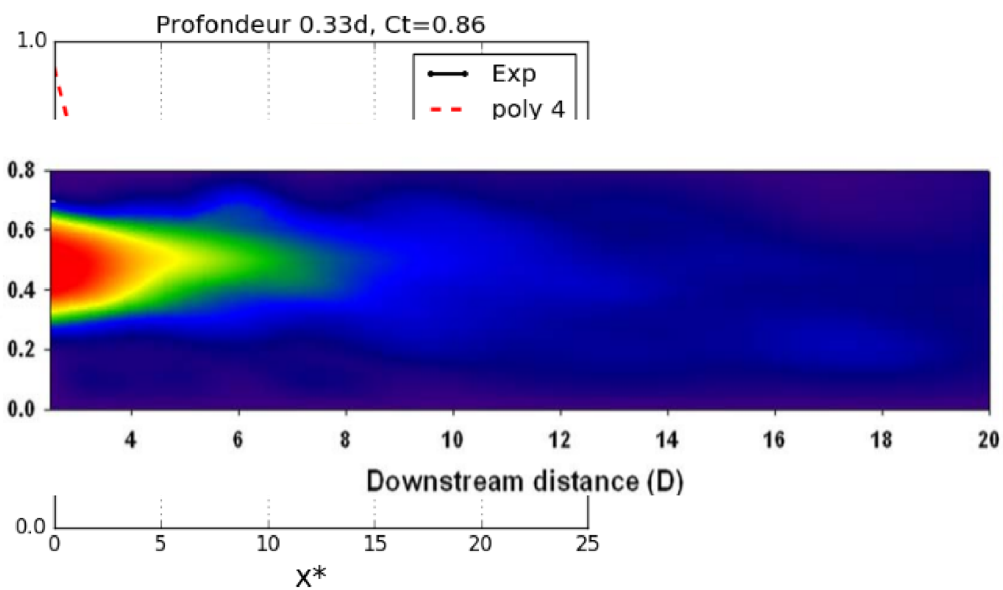
II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

→ New approach to get better flow statistics

$$G_{ij}(t, X_t)\,(U_t - \langle U \rangle) \longrightarrow G_{ij}(t, X_t)\,(U_t - U_{ref})$$

$$U_{ref} = \frac{\alpha}{\alpha + \alpha'} \langle U \rangle + \frac{\alpha'}{\alpha + \alpha'} U_{myers}(x^*, a)$$



$$U_{myers} = U_\infty(1 - U_{deficit} f(z^*))$$

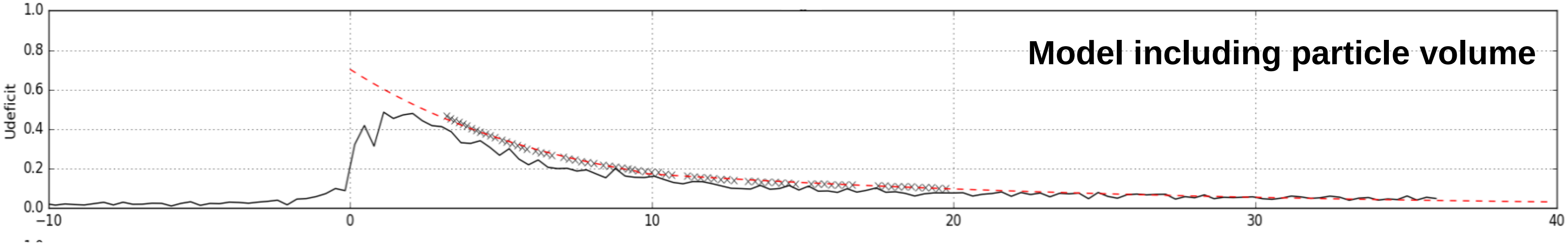
Vertical ponderation

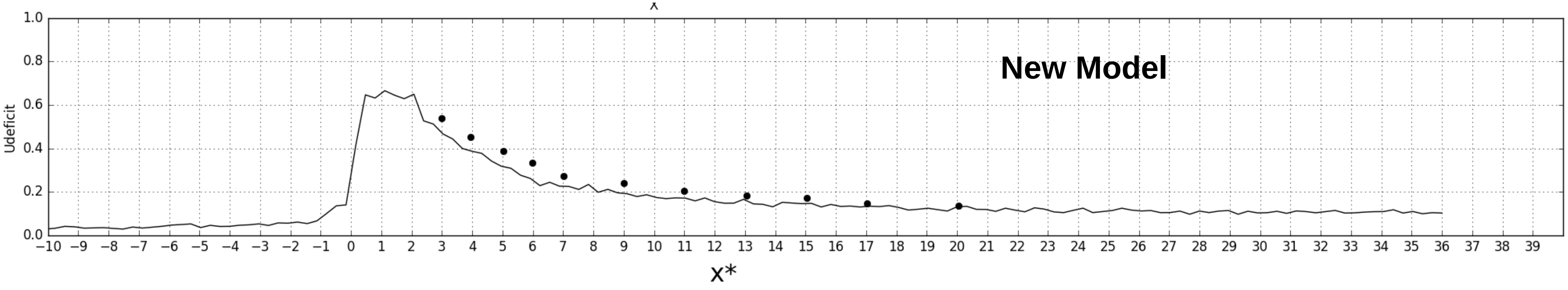
II. Benchmarks

II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

Velocity deficit on the centerline



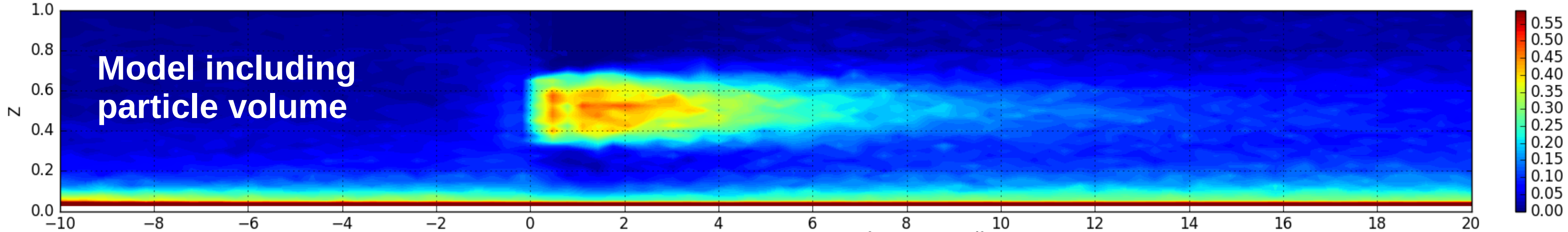


II. Benchmarks

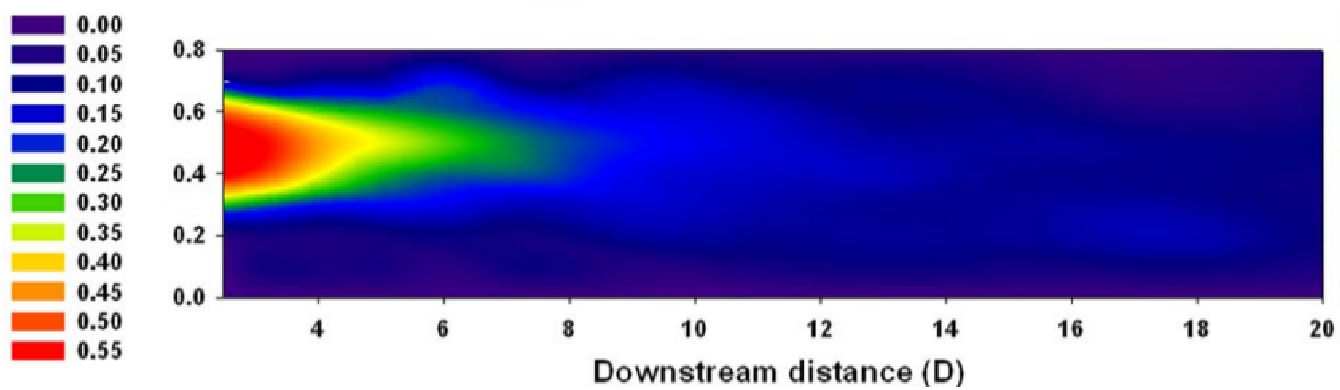
II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

Comparison of Spatial distribution of the velocity deficit



Experimental

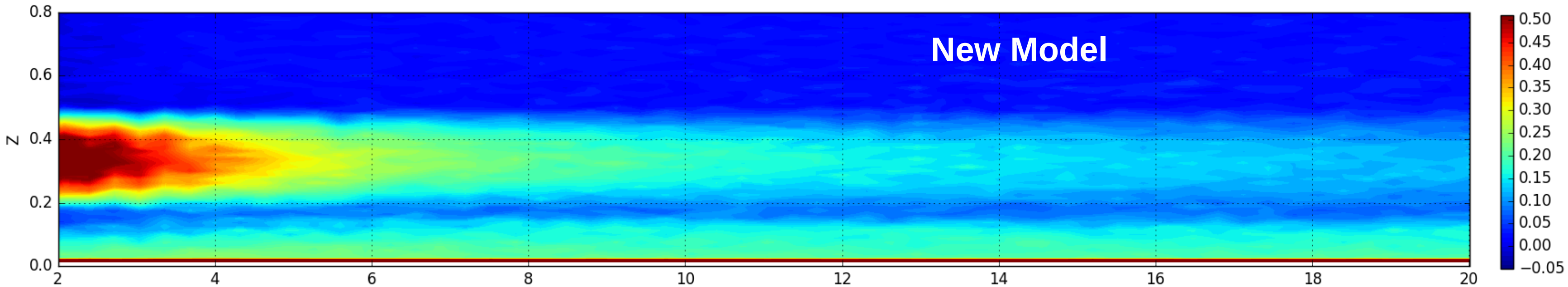


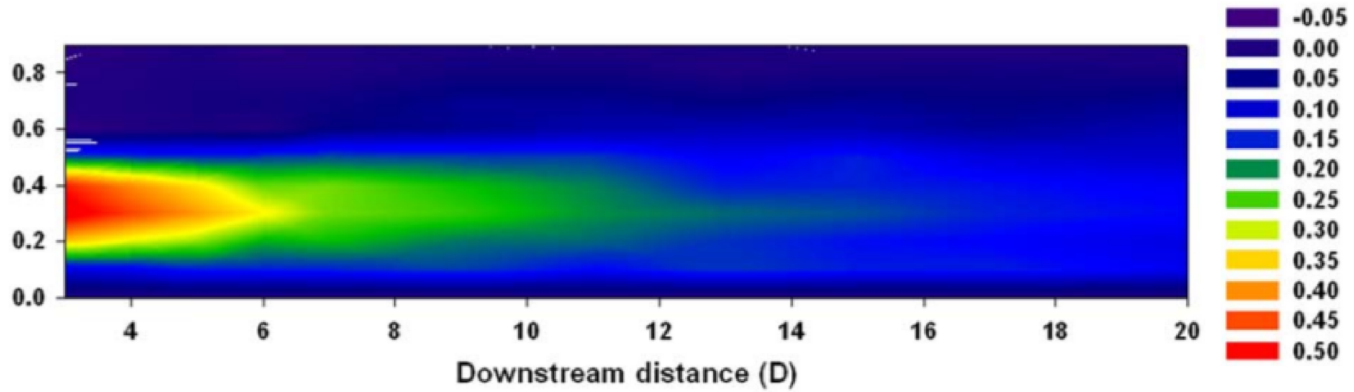
II. Benchmarks

II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

Comparison of Spatial distribution of the velocity deficit





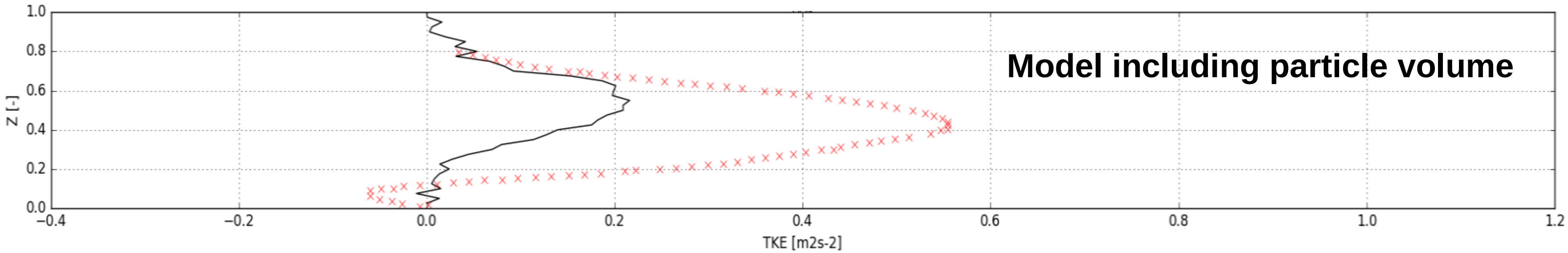
Exp.

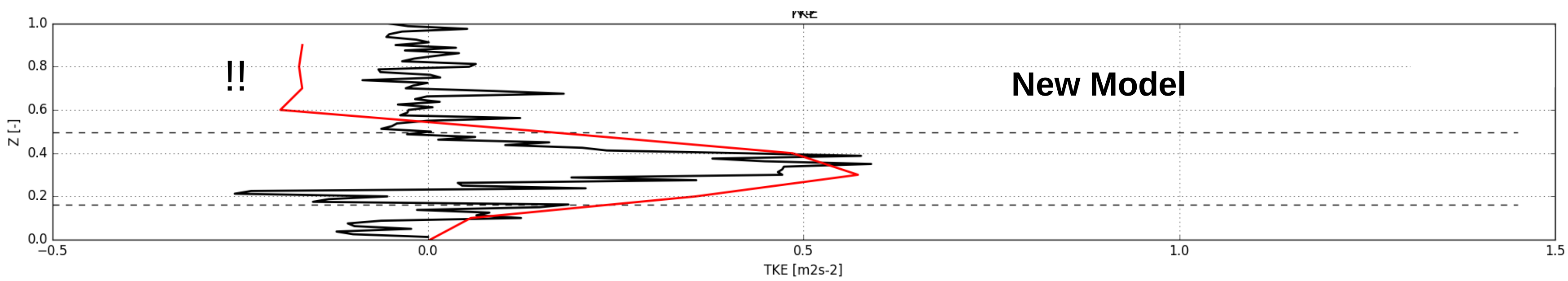
II. Benchmarks

II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

TKE



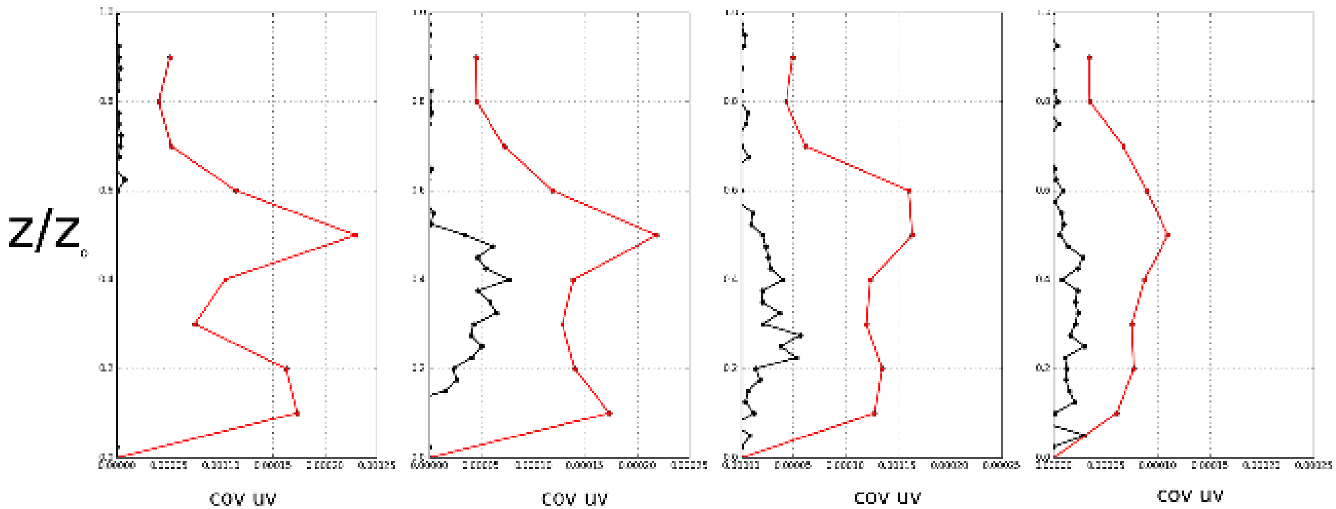


II. Benchmarks

II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)

3D 5D 7D 9D

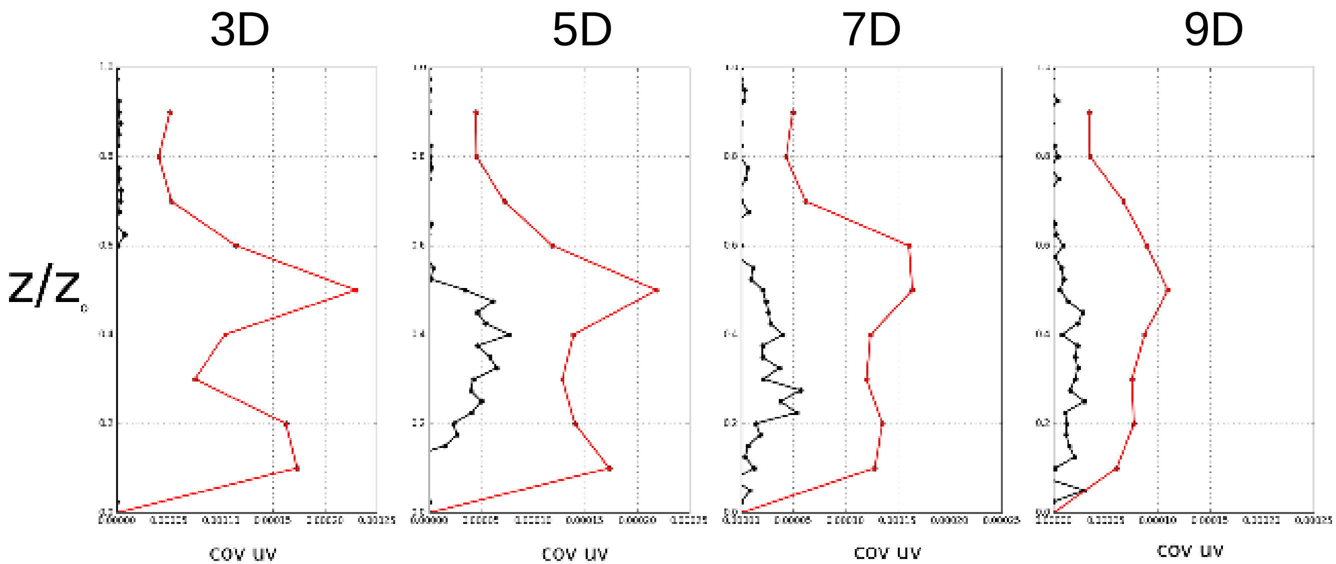


Model including particle volume

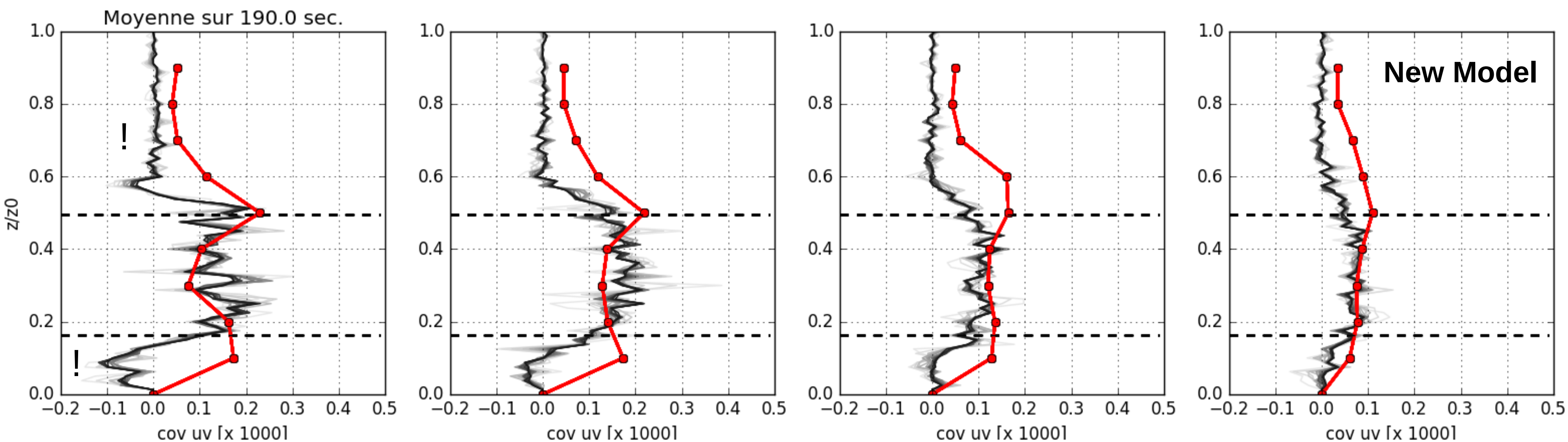
II. Benchmarks

II.3 Turbulence generated downstream turbines

New Model (using calibration from exp. data)



Model including particle volume



III. Conclusion

- Boundary layers quite well represented
  → Both mean velocity field and turbulence
  → However the sub-viscous layer is not included

- Effect of bathymetry
  → Mean fields are well described
  → Some efforts are needed close to the hill

- Porous disk
  → Partially validated
  → Velocity deficit is well described
  → TKE is well predicted far from the interface
  → Covariances have good tendency but lack of accuracy

- Computational times quite reasonable

III. Perspectives

- Improve the porous disks
  → Collaboration with Cristian Escauriaza

- Improve the boundary layer
  → Including the sub-viscous layer

- Propose more complex model of turbines including rotation
  → Collaboration with Hydrotube (Bordeaux)

- Parallelization of the code SDM to reduce computational times

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