Extreme dependence models and Environmental contours for safety assessment

Simulation et Optimisation pour les Energies Marines Renouvelables — Janvier 2018

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Context

- 100-year return value of response is needed for design purpose ;
- Metocean parameters impact is fundamental ;
- Multivariate setting : waves, wind, currents ;
- 100-year return level for each parameter is too conservative ;
- Structure models are too time-consuming to obtain link between metocean variables and responses ;
- Focus here on structure independent methods.

Outline of the talk

1. Case study : tensions in mooring lines

2. Methodology

3. Results

4. Conclusion — Perspectives

Case study : tensions in mooring lines

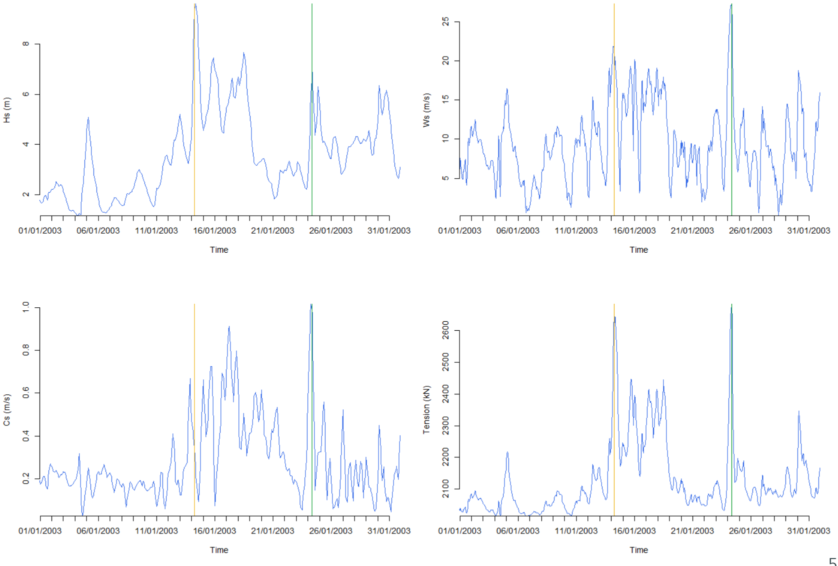
Mooring lines tension

- Data provided by Engie, for Gjoa semi-submersible plateform, 40km West offshore Norway, 100km North of Bergen ;

Source: http://www.energy-pedia.com

- Tensions in the mooring lines available during 20 storms, from 2011 to 2016 at 1Hz with many missing values ;
- Meta-model constructed to reproduce the observed data ;
- Retained parameters : H_s, W_s, C_s and their corresponding directions ;
- Reanalysis databases used to obtain synthetic response of the structure.

Mooring lines tension



Methodology

Extremal Values Modelling : Methodology

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- Hourly mooring line tension, from 1992/10 to 2015/12 ;
- POT modelling :
  - Extraction of storms : events above the 97.5% quantile, separated with at least 1 day ;
  - GPD hypothesis for the 99% quantile excess, parameters estimated with ML ;
  - Computation of the 100-years return level.
- Considered as the Golden Standard to reproduce ;
- Unavailable when designing new structures.

	Threshold	Nb. Obs.	Scale	Shape	100yr r.l.
Tension	2475.76	236	190.01	0.02	3761.36
Hs	7.03	194	1.75	-0.20	13.21
Ws	21.33	305	3.09	-0.18	33.41
Cs	0.88	292	0.20	-0.31	1.46

Table 1: Parameters of the fitted models and estimated return levels

Methodology

Extremal Values Modelling : dependency

Dependence modelling : basic approaches

- Independence : product of cdf ;
- Perfect dependence : 100-years RL occurs in all variables simultaneously ;

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Conditional models

- Standard method (e.g. DNV-GL, IEC-61400-3):
  - Weibull model for W_s ;
  - Weibull model for H_s given W_s ;
  - Log-normal distribution of T_p given H_s ;
- Linear model on the parameters of the distribution ;
- Only 2D ;
- No specific model for extreme, the bulk of the distribution is used.

Gaussian Copula — Nataf Method

- Classical approach in Structural engineering ;
- X (physical space) $\hookrightarrow$ Z (normal space) ;
- Estimation : find Γ s.t.

$$\mathbb{P}(G_\Gamma > \mathbf{u}) = \mathbb{P}(Z_1 >= u_1, Z_2 >= u_2, Z_3 >= u_3)$$

where $G_\Gamma \sim \mathcal{N}(0, \Gamma)$

Extreme value Dependence function

- Thresholds excess X (physical space) $\hookrightarrow$ Y (Frechet space) ;
- Probabilistic results on the joint p.d.f. of Y ;
- A.k.a copula model ;
- Parametric models can be used ;
- Estimation : censored likelihood ;
- Simulation is straightforward.

Conditional Model (Heffernan & Tawn)

- Thresholds excess $\boldsymbol{X}$ (physical space) $\hookrightarrow$ $\boldsymbol{Z}$ (Gumbel space)
- Non-linear regression model fitted using ML and assuming :

$$\boldsymbol{Z}_{-i}|Z_i = \boldsymbol{a}_{-i|i}Z_i + Z_i^{\boldsymbol{b}_{-i|i}}\boldsymbol{\epsilon}_{-i|i}, \text{ for } Z_i > \nu \text{ and } Z_i > \boldsymbol{Z}_{-i}$$

  where:
  - $\boldsymbol{Z}_{-i}$: all variables excluding Z_i ;
  - $\boldsymbol{a}_{-i|i}$ and $\boldsymbol{b}_{-i|i}$: parameters of the fitted pair-wise regression model ;
  - ν : dependency threshold ;
  - $\boldsymbol{\epsilon}_{-i|i} \overset{i.i.d}{\sim} \mathcal{N}(\boldsymbol{\mu}_{-i|i}, \boldsymbol{\sigma}_{-i|i})$.
- Simulation :
  - Draw $\boldsymbol{\epsilon}_{-i|i}$ (empirical, kernel smoothing, Gaussian...) ;
  - Rejection step to respect the empirical ratio of $Z_i > \boldsymbol{Z}_{-i}$;
  - Transform back to original scale $Y_i = \exp(-F_i(Z_i))$.

Methodology

Environmental contours

Environmental contours

- Environmental contours : curves along which N-year levels of response will lie ;
- Contours are independent of the structure ;
- Hypothesis :
  - the ruin of the structure occurs in a single point in the space ;
  - The limit state is well approximated by an hyperplane.

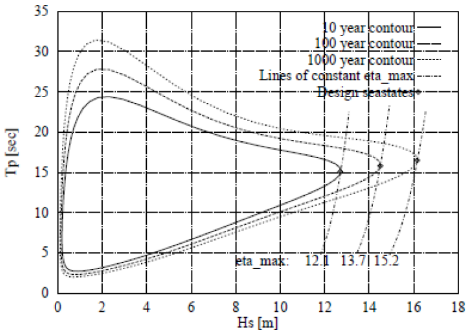

Figure 1: Example of Hs-Tp contours and extreme wave crest (eta) as response (source: Wintershein-et-al 1993)

Huseby method

- Direct Monte Carlo simulations ;
- Works in the original space ;
- For any direction θ, one can estimate the hyperplane with p-probability of being exceeded ;
- The contour is the curve which tangents all these straight lines ;

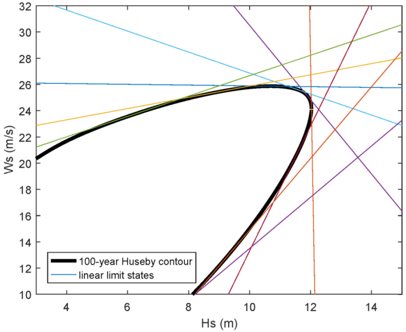

Figure 2: Huseby contour method

Results

Results

Method	Tension	Rel. Err.	Hs	Ws	Cs
Meta model	3761.36	NA	NA	NA	NA
Independence	3536.22	-5.99	12.93	25.87	1.10
Perfect dependence	4152.32	10.39	13.21	33.41	1.46
Nataf	3911.71	4.00	13.02	32.11	1.28
Logistic model	3994.31	6.19	12.98	32.37	1.40
Conditional extremes	3627.88	-3.55	11.97	31.00	1.27

Table 2: Comparison of the methods for the estimation of 100-years return level.

Results

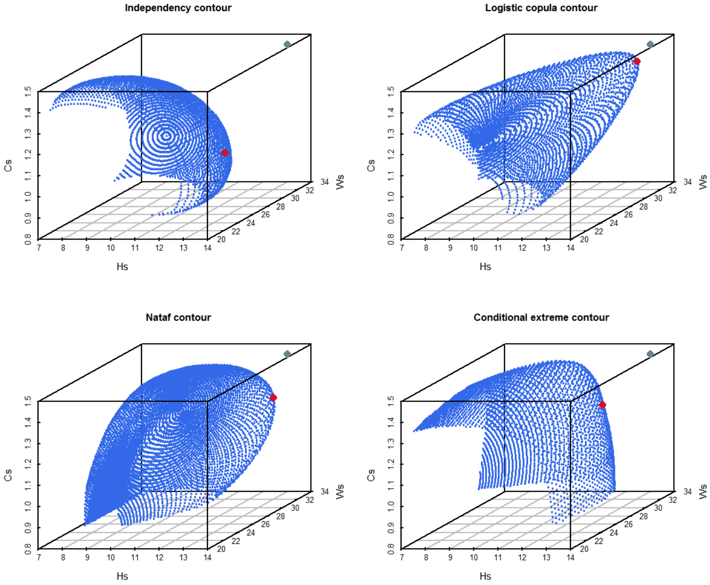

Figure 3: 3D contour and design points (red).

Conclusion — Perspectives

Conclusion — perspectives

- Comparison of multivariate methods to obtain extremal environmental conditions ;
- Both Parametric copula and Heffernan&Tawn performs well in 2D & 3D ;
- Procedure leading to realistic and less conservative design point ;

Future works perspectives :

- Higher dimensions and directional aspect ;
- Non-exchangeable model for copula ;
- Contours for coupled intensity variable and covariate (e.g. H_s and T_p) ;